Textbook Reading: Chapters 5, 6, 7 **Due Date:** Friday, February 5, 2021 by 11:59pm

Instructions

- Upload a copy of your assignment (pdf format) to the Crowdmark link you've received via email.
- *Correctness, Clarity, & Conciseness* of presentation are reflected in the grading.
- Collaborative discussion on the assignment in encouraged, but the write-up should reflect you own understanding & results. Acknowledge colleagues, TA, or other assistance you received.

Questions

1. **Swap Puzzle arrangements and moves in cycle notation.** The following diagram shows a sequence of moves that have been applied to a scrambling of the tiles in Swap.

	1	2	3	4	5	6	7	8
start	5	4	2	8	1	3	6	7
$\xrightarrow{\tau_1}$	5	2	4	8	1	3	6	7
$\xrightarrow{\tau_2}$	1	2	4	8	5	3	6	7
$\xrightarrow{\tau_3}$	1	2	4	8	5	3	7	6

Do the following:
(a) Express the starting position α as a permutation in cycle notation.
(b) Express each move τ_i as a 2-cycle.
(c) Express the whole move sequence $\tau_1\tau_2\tau_3$ as a permutation in cycle notation.
(d) Express the final position β as a permutation in cycle notation and show that the following equation is true $\alpha\tau_1\cdots\tau_n = \beta$.

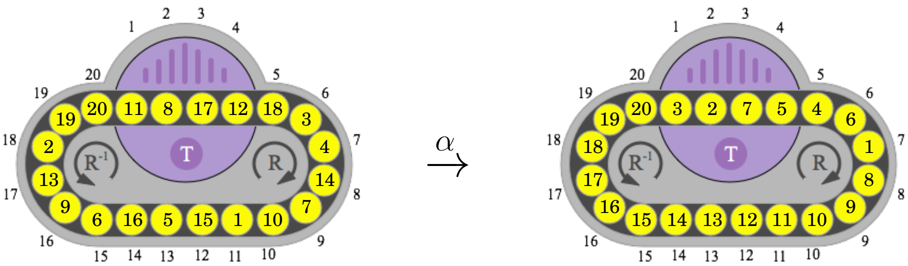
2. **Decomposing a permutation into 3-cycles.** Write the permutation $\alpha = (1\ 2)(3\ 4)$ as a product of 3-cycles.
(Hint: Solve the corresponding Swap puzzle, under the variation where the legal moves are now 3-cycles, and write down the permutations representing your sequence of moves.)

3. **15-Puzzle position into cycle notation.** Express the scrambling of the 15-puzzle as a permutation in cycle form.

[1] 2	[2] 6	[3] 5	[4] 4
[5] 1	[6] 15	[7]	[8] 14
[9] 8	[10] 9	[11] 7	[12] 3
[13] 13	[14] 11	[15] 12	[16] 10

4. **15-Puzzle arrangements from cycle notation.** For the permutation $(1\ 10\ 5\ 15)(2\ 4\ 8)(6\ 7\ 12)$, draw the corresponding scrambling of the tiles on the 15 puzzle.
(A 15-puzzle templates is available as .png files from the Assignments page.)

5. **Oval Track Puzzle move sequence in cycle notation.** For the Oval Track puzzle in the diagram below do the following.
(a) Express the position of the puzzle configuration on the left as a permutation β in cycle form.
(b) Express the move sequence α as a permutation in cycle form.
(c) Verify that the permutation representing the position on the right is equal to the product of β and α.



6. For each of the following permutations, in cycle form, write it as a product of 2-cycles. State whether the permutation is *even* or *odd*.

   (a) $(2\ 4\ 7\ 9)(3\ 5\ 8)$ (b) $(1\ 2\ 3\ 4\ 5)(6\ 7\ 8\ 9\ 10)$

7. **The parity of 15-puzzle scrambles.** For each of the following arrangements of the 15-puzzle determine the parity of the corresponding permutation.

1	2	3	4
5	6	7	8
14	10	12	15
9	13	11	

(a)

13	9	5	3
2	1	7	10
	15	14	8
12	11	6	4

(b)

8. Show each of the following.

   (a) The product of two even permutations is an even permutation.

   (b) The product of two odd permutations is an even permutation.

   (c) The product of one even permutation and one odd permutation is an odd permutation.

9. (a) If α is even, prove that α^{-1} is even.
   (b) If α is odd, prove that α^{-1} is odd.
   In other words, show that α and α^{-1} have the same parity.

10. Let $\alpha, \beta \in S_n$. Prove that α and $\beta^{-1}\alpha\beta$ have the same parity.

11. (a) Give an example of an even permutation with even order.
    (b) Give an example of an odd permutation with even order.
    (c) Show that a permutation with odd order must be an even permutation.