Textbook Reading: Chapters 9 **Due Date:** Tuesday, February 23, 2021 by 11:59pm

Instructions

- Upload a copy of your assignment (pdf format) to the Crowdmark link you've received via email.
- *Correctness, Clarity, & Conciseness* of presentation are reflected in the grading.
- Collaborative discussion on the assignment in encouraged, but the write-up should reflect you own understanding & results. Acknowledge colleagues, TA, or other assistance you received.

Questions

1. **Solvability of the 15-puzzle:** Use Theorems 9.1.1 and 9.1.2 to determine which of the following arrangements of the 15-puzzle are solvable and which are unsolvable.

5	1	2	3
9	6	8	4
13	10	7	12
14	15	11	

(a)

1	2	3	4
	9	11	6
14	13	15	8
5	12	10	7

(b)

8	6	3	
2	7	5	14
9	1	15	12
10	13	11	4

(c)

2. **Solving the 15-puzzle:** The following arrangements of the 15-puzzle are solvable. Write down a sequence of moves that solves each puzzle. (Pieces that need to be solved are indicated in red.)

   How to write the moves for your solution: Use the first letter of the words *(u)p, (d)own, (l)eft, (r)ight*, to indicate the direction the next tile is to be moved. This will make is easy for the grader to try your move sequence on the puzzle to see if your sequence of moves solves it.
   Use the online puzzle: `http://www.sfu.ca/~jtmulhol/math302/puzzles-15.html`

1	2	3	4
5	6	7	8
13	**9**	11	12
10	14	15	

(a)

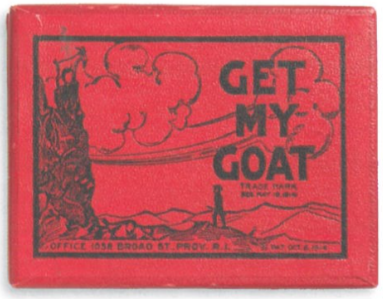
11	2	3	4
5	**1**	7	8
9	10	**6**	12
13	14	15	

(b)

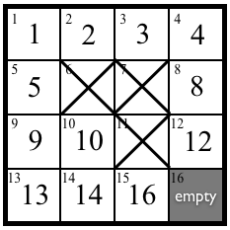
1	2	3	4
5	6	7	8
13	10	**12**	**11**
9	14	15	

(c)

3. The **Get My Goat Puzzle** was patented in 1914. The problem is to get the goat inside the fenced-in area, after removing the marked block. This basically requires a swap of the block with the picture of the goat's head and the block adjacent to it. Since the solution is to seemingly *swap two pieces* then at first glance it would seem this puzzle is not solvable by analogy with the 15-puzzle. However, this puzzle is solvable. Provide an explanation as to why this puzzle is solvable (use the results we developed for the 15-puzzle).
   Play the puzzle online:
   `http://www.sfu.ca/~jtmulhol/math302/applets/get-my-goat/get-my-goat.html`



4. The following board shows a variation of the 15 puzzle where boxes 6, 7 and 11 are obstacles. That is, these boxes are "out-of-play" and cannot be used. We can still ask the question as to which permutations of the tiles are solvable. Show that, just like the original 15 puzzle, Theorems 9.1 and 9.2 remain true.



(Hint: For simplicity just focus on permutations leaving the empty space in its home location. Use the two-by-two square of four boxes to generate all 3-cycles: first show you can obtain one 3-cycle, then use techniques for modifying a 3-cycle to produce any other 3-cycle.)

Applet: You can use this applet to explore the puzzle, just keep in mind the tiles with X's should not be allowed to move:
`http://www.sfu.ca/~jtmulhol/math302/applets/fifteen-puzzle-blockage/15-puzzle.html`