MATH 304

Textbook Reading: Chapters 13, 14

Due Date: Friday, March 12, 2021 by 11:59pm Week Date Sections Part/ References Notes/Speaker Topic/Sections from FS2009 Questions, 1.2, 1.3 Symbolic methods Combinatorial Structures 1.4. 1.5. 1.6 Unlabelled structures All about commutators. 11.1, 11.2, 11.3 Labelled structures I 3 21 Handout #1 (a) Let $\alpha, \beta \in S_n^{\text{lel}}$ Show that the commutator $[\alpha, \beta]$ is an even permutation. 4 (b) Show that if grink torial then gand hroommute. 5 (c) Show that $[\mathfrak{g} h] \Pi^1 =$ Parameters [h,g]12 6 IV.1, IV.2 self-study) Multivariable GFs 2. Let $\alpha, \beta \in \mathcal{S}_n$. Complex Analysis Analytic Methods (a) If $\max_{\alpha \in \mathcal{A}} (\alpha) = \beta$, what is $(\alpha, \beta) = \beta$, what is $(\alpha, \beta) = \beta$, what is $(\alpha, \beta) = \beta$, what is 8 Nov 2 the permutation of the per Asst #2 Due Asymptotic methods (b) If $\operatorname{vmov}(\alpha)$ and $\operatorname{vmov}(\beta)$ have two locations (elements) in common (i.e. $|\operatorname{mov}(\alpha) \cap \operatorname{mov}(\beta)| = 2$), what is 10

- the largest $|mov([\alpha, \beta])|$ can be duction to Prob. 12 Mariolys
- 13. Do exercises 14, 15 in Chapter 19 to the textbook mi You do not need to hand anything in for these questions, I true try out did them Di Afternally you aven in this course because you wanted to learn about the mathematics of the Rubik's cube so xou wouldn't deny yourself a valuable, and fun, learning opportunity
- ;-) Playing with these exercises should give you some insight into how to construct your own puzzle moves. 12 Try to greate some of your own using commutators
- 14. Flip 2 adjacent edges. In this exercise you will construct a move to produce a double edge flip in the up layer, as shown in Figure 1b. Asst #3 Due
- 14





(b) Double edge flip

(a) Slice move M_R .

Figure 1: Diagrams for exercise 4

Let M_R denote the "slice move" which consists of rotating the middle slice, parallel to the R face, in the clockwise direction, from the perspective of the R face. See Figure 1a. Consider the move sequence

$$\alpha = M_R^{-1} D M_R D^{-1} M_R^{-1} D^2 M_R$$

- (a) Verify α flips the edge in the *uf* position, and fixes everything else in the *up* layer.
- (b) Since α only affects one cubies in the up layer, what would be a good choice for move β , so that the commutator $[\alpha,\beta]$ affects only two edge cubies? With your choice of β can you predict the effect of $[\alpha, \beta]$ on the cubies?
- (c) Perform the move $[\alpha, \beta]$ and verify your prediction from the previous part. The move sequence should produce the double edge flip as shown in Figure 1b (or similar).

Sections Part/ References Topic/Sections Notes/Speaker Week Date 5. Flip 2 for presenter edges. Find moves α and β so that the commutator $[\alpha, \beta]$ flips two opposite edges (as, shown in the diagram below), and fixes everything else. (Hint: Modify the moves in the previous exercise.) Structures 2 Unlabelled structures FS: Part A.1, A.2 Comtet74 3 21 11.1, 11.2, 11.3 Labelle ' Handout #1 (self study) 4 28 11.4, 11.5, 11.6 Labelle Combinatorial Combi III.1, III.2 5 Oct 5 parameters Param FS A.III 6 12 IV.1, IV.2 (self-study) Multiva 7 19 IV.3, IV.4 Comple Analytic Methods FS: Part B: IV, V, VI 26 Singula 8 Appendix B4 IV.5 V.1 Stanley 99: Ch. 6 Nov 2 Asymptotic methods Handout #1 (self-study) 0 VI 1 Sophie 16. Building a corner 3-cycle. In this exercise we build a 3-cycle of corners in the up layer. (a) Verify the the conjugate ULUminil brings one new corner cubie into the right face. 11 (b) Since ULU-Random structure ne Riscornier cubie into the right face this makes a good candidate to form a commutator with cR^{-1} . Verify the commutator $[U_{s}LU^{-1}, R^{-1}]$ moves the corner cubies as indicated in 23 the diagram represented in places of discrete also given notationally as follows 12 Continuous Limit Laws Marni 25 IX.4 $ruf \mapsto rbu, \quad rbu \mapsto ulf.$ Quasi-Howers and ruf,Sophie 13 30 IX.5 Gaussian limit laws 14 Dec 10 Presentations

- 7. For each of the following pairs of permutations state whether they are conjugate in S_{10} . That is, determine whether there exists a $\gamma \in S_{10}$ so that $\beta = \gamma^{-1} \alpha \gamma$. If they are conjugate give an example of a permutation $\gamma \in S_{10}$ so that $\beta = \gamma^{-1} \alpha \gamma$.
 - (a) $\alpha = (1\ 2\ 3\ 4\ 5\ 6\ 7)(8\ 9\ 10), \qquad \beta = (1\ 5\ 8)(2\ 6\ 3\ 7\ 4\ 10\ 9)$ (b) $\alpha = (1\ 5\ 8)(2\ 6)(3\ 7\ 4), \qquad \beta = (1\ 2)(7\ 3)(8\ 9\ 10)$
- 8. Let G be a group. Prove that every conjugate of a commutator is a commutator by showing that $g[a,b]g^{-1} = [gag^{-1}, gbg^{-1}]$ for all $a, b, g \in G$.
- 9. Show that for $g_1, g_2, h, h_1, h_2 \in G$ the following hold.

(a)
$$(g_1g_2)^h = g_1^h g_2^h$$

(b) $g^{h_1h_2} = (g^{h_1})^{h_2}$

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(Recall, the exponential notation g^h is shorthand for conjugation $h^{-1}gh$. These two properties indicate why this shorthand notation is used.)