Redo all homework assignments again to prepare for the midterm. Below are questions for additional practice.

1. Consider the following two arrangements of the 15-puzzle, one is solvable and the other is not. Determine which puzzle is not solvable (justify why using Theorems in the course). For the solvable puzzle Write down a sequence of moves that solves the puzzle. (Write the solution by using the first letter of the words *(u)p, (d)own, (l)eft, (r)ight*, to indicate the direction the next tile is to be moved.)

^{1} 6	^{2} 4	^{3} 1	^{4} 14
^{5} 5	^{6} 7	^{7}	^{8} 11
^{9} 13	^{10} 2	^{11} 12	^{12} 3
^{13} 8	^{14} 10	^{15} 15	^{16} 9

(a)

^{1}	^{2} 1	^{3} 2	^{4} 3
^{5} 5	^{6} 6	^{7} 7	^{8} 4
^{9} 9	^{10} 13	^{11} 11	^{12} 8
^{13} 14	^{14} 10	^{15} 15	^{16} 12

(b)

2. Does A_8 contain and element of order 10? If so, give an example. If not, show why.
3. Find permutations α and β so that $\text{ord}(\alpha) = 2$, $\text{ord}(\beta) = 2$, and $\text{ord}(\alpha\beta) = 6$.
4. Suppose α is a 15-cycle. For which integers k between 2 and 15 is α^k also a 15-cycle?
5. Write down a solution to the 3-cycle swap puzzle when the initial configuration is $\beta = (1\ 3\ 5)(2\ 4\ 7\ 6\ 8)$.
6. Show that the set $\{\alpha \in S_n \mid \alpha^5 = \epsilon\}$ has cardinality which is of the form $4k + 1$.
7. Determine the maximum order of an element in S_{10}. Determine the maximum order of an element in A_{10}.
8. Give an example of an element in A_{20} which contains at least one 3-cycle, and at least one 4-cycle in its disjoint cycle form, and has order 60.
9. How many elements of A_7 have order 3?
10. Starting with the configuration on the left a sequence of moves is applied given by

$$\alpha = (1\ 6\ 2)(3\ 5).$$

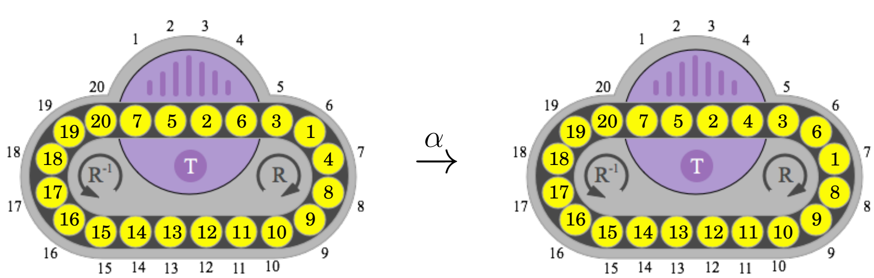
Fill in the board on the right with the configuration that results.

^{1} 6	^{2} 3	^{3} 5	^{4} 4	^{5} 1	^{6} 2

$\xrightarrow{\alpha}$

^{1}	^{2}	^{3}	^{4}	^{5}	^{6}

11. **Oval Track Puzzle move sequence in cycle notation.**
For the Oval Track puzzle in the diagram below do the following.
   (a) Express the position β of the puzzle configuration on the left as a permutation in cycle form.
   (b) Express the move sequence α as a permutation in cycle form.
   (c) Express the position γ on the right as a permutation in cycle form, and show that $\beta\alpha = \gamma$.



12. **Permutations: decompositions into 2-cycles of the form $(m, m+1)$:**

    Show that every permutation in S_n can be expressed as a product using only the 2-cycles:

$$(1,2), (2,3), (3,4), \ldots, (m, m+1), \ldots, (n-1, n)$$

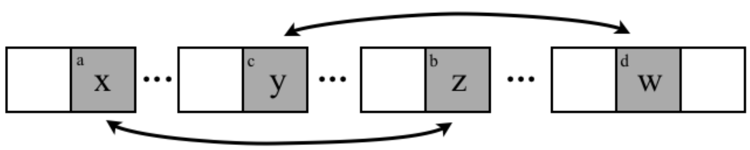
13. **Permutations: decompositions into 2-cycles of the form $(1\ m)$:**

    We know that every permutation in S_n can be expressed as a product of 2-cycles. Show the stronger result that every permutation in S_n can be expressed as a product using only 2-cycles of the form $(1\ m)$, where $2 \leq m \leq n$.

    (This is equivalent to showing that every permutation is obtainable on the Swap puzzle where the only legal move is to swap the contents of any box with box 1. You may use the Swap puzzle to investigate this statement, but the argument you present should be described in terms of permutations.)

14. In this question you are to verify a solvability criteria for a variation of the Swap puzzle on n objects, where $n \geq 5$.

    Variation: Legal moves are products of disjoint 2-cycles: $(a, b)(c, d)$, where $1 \leq a, b, c, d \leq n$. In other words, a move consists of picking any two pairs of boxes, say pair $\{a, b\}$ and pair $\{c, d\}$, and swapping the contents of boxes a and b, as well as swapping the contents of c and d. See diagram for an example of such a move.



    Prove the following solvability criteria for this puzzle.

    Solvability Criteria: For the Swap puzzle with $n \geq 5$, with legal moves as described above, an arrangement of the tiles is solvable if and only if the corresponding permutation is *even*.