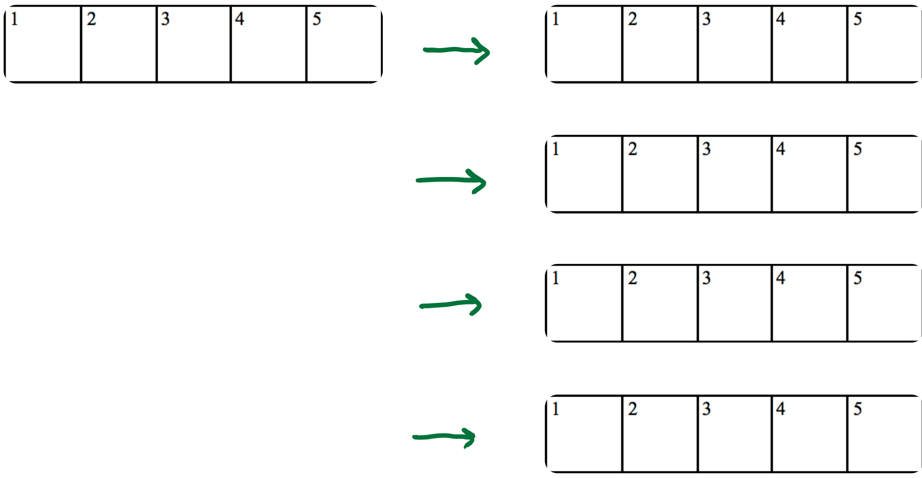
# Permutations: Products of 2-cycles (transpositions)

Ex: Write the permutation $\beta = (1\ 5\ 3\ 4\ 2)$ as a product of 2-cycles.
(Hint: Solve the corresponding swap puzzle with $\beta$ as the initial configuration, and keep track of your moves.)




**Theorem 6.2.1 — Product of 2-Cycles.** Every permutation in $S_n, n > 1$, can be expressed as a product of 2-cycles.

k-cycle into 2-cycles:

Ex:

In general,

$(a_1\ a_2 \ldots a_k) =$

Ex: Express $(1\ 2\ 3)$ as a product of 2-cycles.

Ex: Express $\alpha = (1\,5\,4)(2\,8\,3\,6)(7\,9)$ as a product of two cycles.

Proof of thm 6.2.1:

## Solvability of Swap:

A permutation $\alpha$ is obtainable as a position of the swap puzzle iff there is a sequence of moves (2-cycles) $\tau_i$ taking $\varepsilon$ to $\alpha$.

$$\alpha = \tau_1 \tau_2 \cdots \tau_K$$

This is equivalent to saying

$\alpha$ is a "legal" position of swap $\iff$ $\alpha$ is a product of 2-cycles

By Thm 6.2.1 it follows every permutation is a legal configuration (i.e. is solvable).

**Corollary 6.3.1** The Swap puzzle, where the legal moves consist of swapping contents of any two boxes, is solvable from any configuration. In other words, all permutations in $S_n$ can be obtained in the Swap puzzle on $n$-objects.