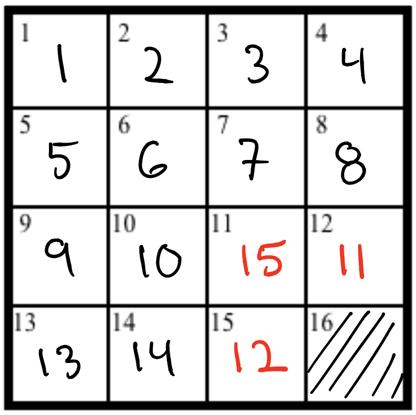
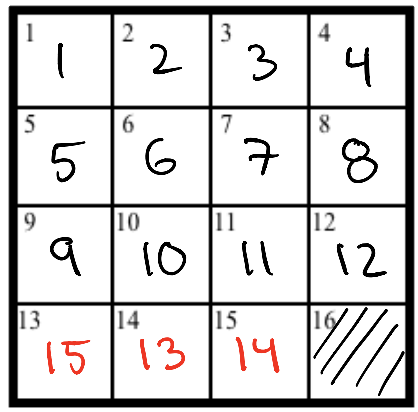
Chapter 14 - Conjugates

Creating new moves from old ones:

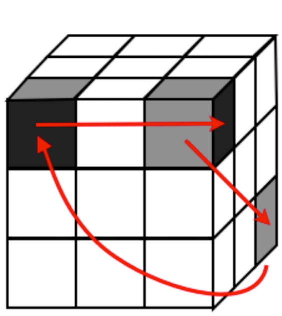


modify to create



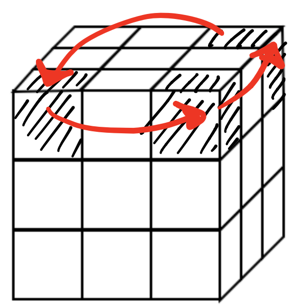
3-cycle: (11 12 15)

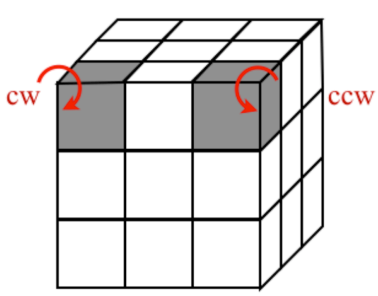
3-cycle: (13 14 15)



$[LD^2L^{-1}, U]$

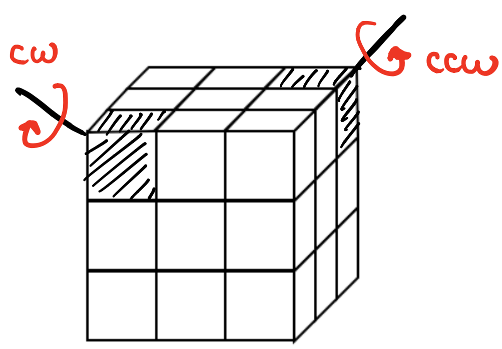
modify to create





$[LD^2L^{-1}F^{-1}D^2F, U]$

modify to create



Conjugates :

Definition 14.1.1 If g, h are two elements of a group G, then we call the element

$$g^h = h^{-1}gh$$

the **conjugate** of g by h.

Note : $g^h = g \iff$

Definition 14.1.2 We say that two elements $g_1, g_2 \in G$ are **conjugate** (in G) if there is an element $h \in G$ such that $g_2 = g_1^h$.

The set of all elements in G that are conjugate to g is called the **conjugacy class of** g and denoted by $\mathrm{cl}(g)$:

$$\mathrm{cl}(g) = \{x^{-1}gx \mid x \in G\}.$$

Conjugation in S_n :

Lemma 14.1.1 — Conjugation preserves cycle structure. Let α, β be any permutation in S_n, and suppose $\alpha(i) = j$. Then $\alpha^\beta = \beta^{-1}\alpha\beta$ sends $\beta(i)$ to $\beta(j)$:

$$(\alpha^\beta)(\beta(i)) = \beta(j).$$

Moreover, if α has cycle structure

$$\alpha = (a_1\ a_2\ \ldots\ a_{k_1})(b_1\ b_2\ \ldots\ b_{k_2})\cdots(c_1\ c_2\ \ldots\ c_{k_m})$$

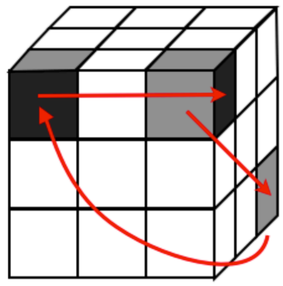
then α^β has the same cycle structure

$$\alpha^\beta = (\beta(a_1)\ \beta(a_2)\ \ldots\ \beta(a_{k_1}))(\beta(b_1)\ \beta(b_2)\ \ldots\ \beta(b_{k_2}))\cdots(\beta(c_1)\ \beta(c_2)\ \ldots\ \beta(c_{k_m}))$$

Proof :

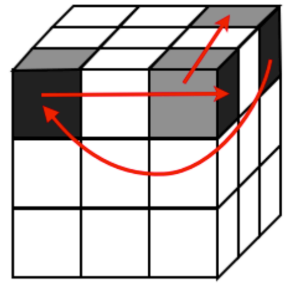
Modifying Puzzle Moves with Conjugates :

Rubik's Cube :



Conjugate by B

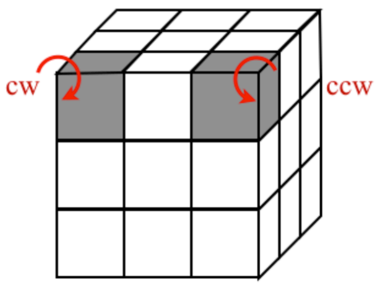
to create



(a) 3-cycle of corner cubies by commutator $LD^2L^{-1}ULD^2L^{-1}U^{-1}$

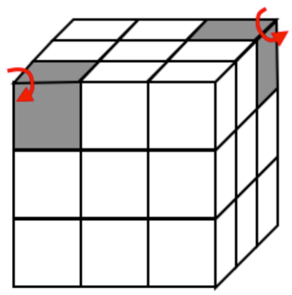
(b) conjugation of commutator by B

Figure 14.1: cycling 3 corner cubies



Conjugate by R

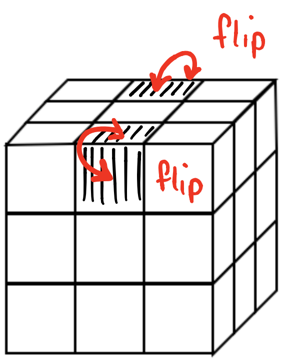
to create



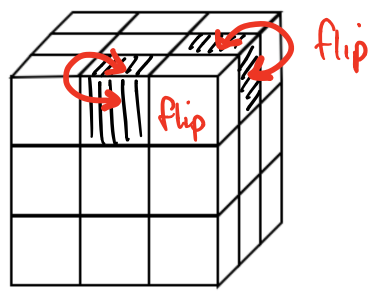
(a) corner twist commutator $xyx^{-1}y^{-1}$

(b) conjugation of commutator by R

Figure 14.2: twisting 2 corner cubies



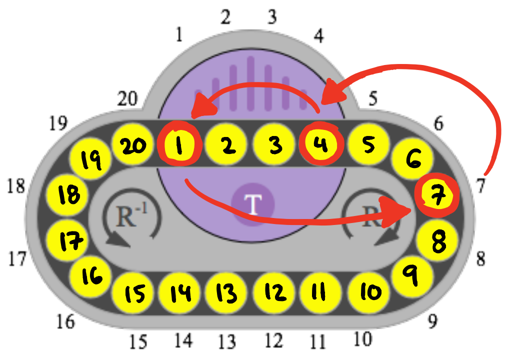
Conjugate by



Suppose X is a move that flips uf and ub

A move that flips uf & ur is

Oval Track



3-cycle

$\gamma = [R^{-3}, T]^2 = (1\ 7\ 4)$

Lets create the 3-cycle $(1\ 2\ 3)$:

Step ① move 1,2,3 to 1,4,7 in any way. Call this β^{-1}.
note: " 1 chases 2 "

Step ② Apply $\gamma^{-1} = (1 4 7)$

Step ③ Apply β

Result: $\beta^{-1}\gamma^{-1}\beta = (\beta(1)\ \ \beta(4)\ \ \beta(7)) = (1\ 2\ 3)$.