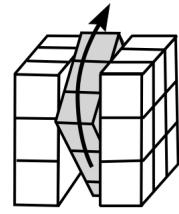


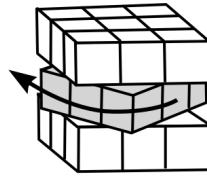
Chapter 19 - Rubik's Cube Beginnings

Notation & Terminology:

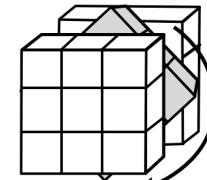
Slice Moves



$$M_R = M_L^{-1}$$

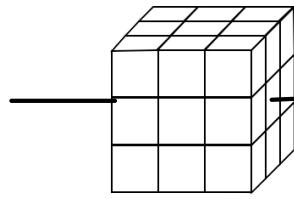


$$M_U = M_D^{-1}$$

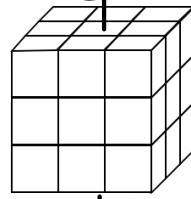


$$M_F = M_B^{-1}$$

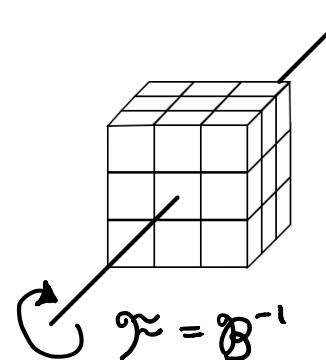
Whole Cube Moves:



$$R = L^{-1}$$



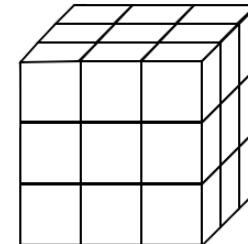
$$U = D^{-1}$$



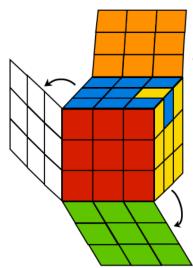
$$F = B^{-1}$$

Facet Notation:

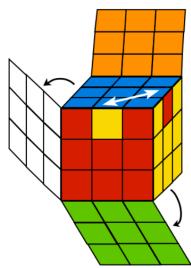
	ulb ub ubr		
	ul U ur		
	ufl uf urf		
lbu lu luf	flu fu fur	rfu ru rub	bru bu bul
lb L lf	fl F fr	rf R rb	br B bl
ldb ld lfd	fdl fd frd	rdf rd rbd	bdr bd bld
	dlf df dfr		
	dl D dr		
	dbl db drb		



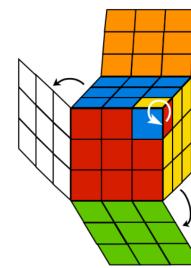
Impossible Moves:



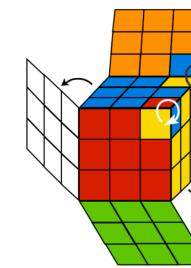
(a) single edge flip



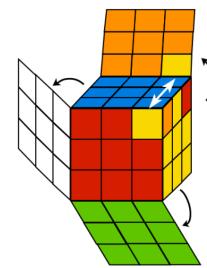
(b) edge swap



(c) single corner twist



(d) double corner twist in same direction

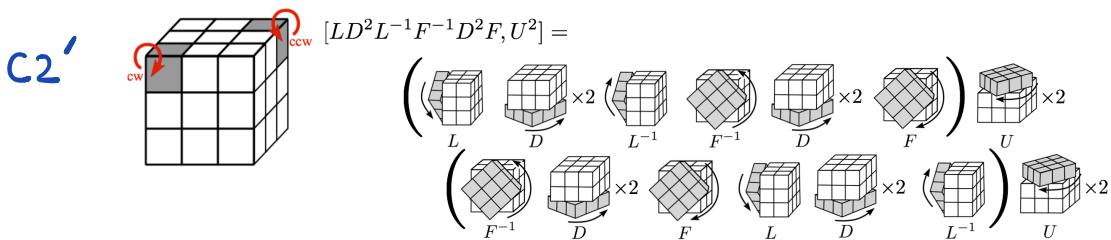
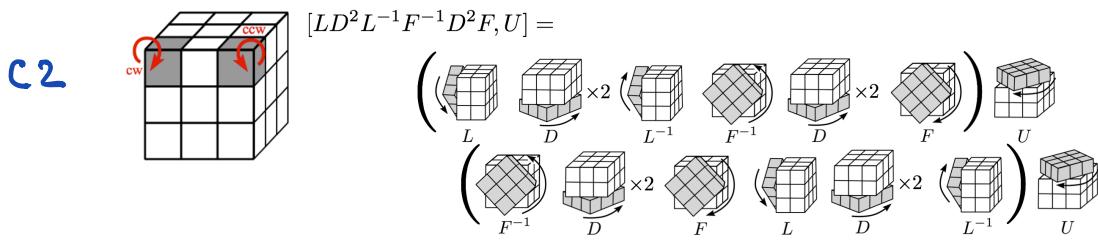
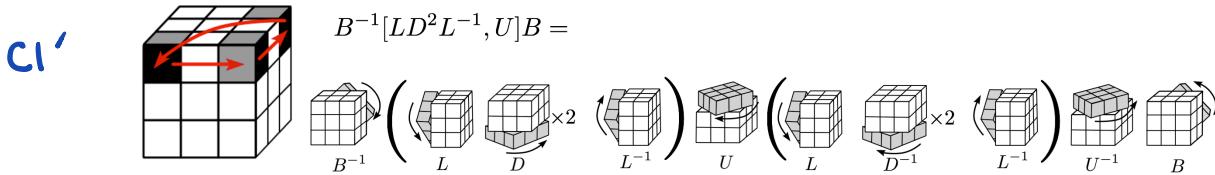
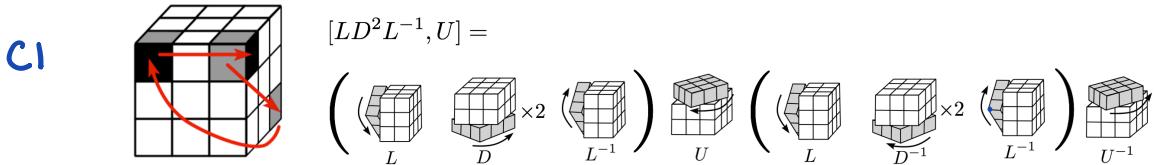


(e) corner swap

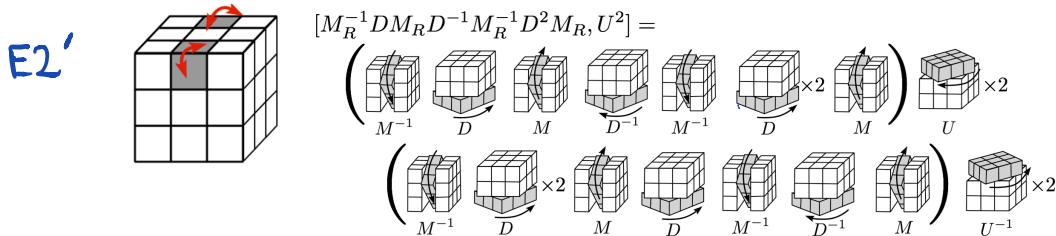
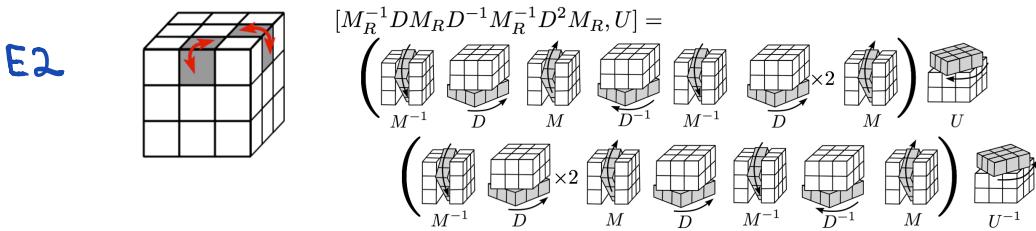
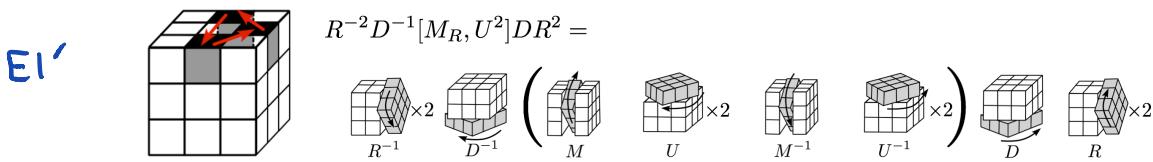
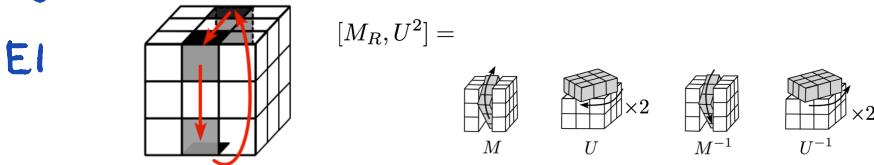
Figure 19.6: Five different configurations that are impossible to achieve.

Catalog of Useful Moves:

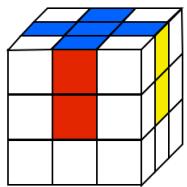
Corner Moves :



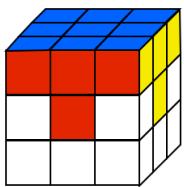
Edge Moves :



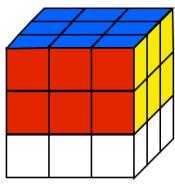
Strategy For Solution :



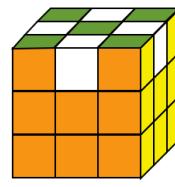
Step 1



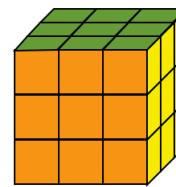
Step 2



Step 3



Step 4



Step 5

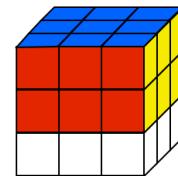
Step 1 & 2 : Complete the up layer

- first solve the corners

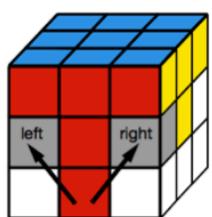
- then solve the edges

No special moves needed here, just move pieces around in the D layer and bring up to U layer using L, F, F⁻¹, R⁻¹ "elevators".

Step 3: Solve edges in the middle layer



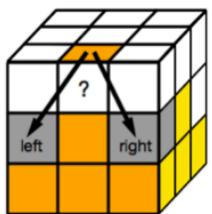
Using Z-commutators :



$$\text{left: } \left(\begin{array}{c} \text{cube} \\ D \\ \text{cube} \\ L \end{array} \right) \left(\begin{array}{c} \text{cube} \\ D^{-1} \\ \text{cube} \\ L^{-1} \end{array} \right) \left(\begin{array}{c} \text{cube} \\ D^{-1} \\ \text{cube} \\ F^{-1} \end{array} \right) \left(\begin{array}{c} \text{cube} \\ D \\ \text{cube} \\ F \end{array} \right)$$

$$\text{right: } \left(\begin{array}{c} \text{cube} \\ D^{-1} \\ \text{cube} \\ R^{-1} \end{array} \right) \left(\begin{array}{c} \text{cube} \\ D \\ \text{cube} \\ R \end{array} \right) \left(\begin{array}{c} \text{cube} \\ D \\ \text{cube} \\ F \end{array} \right) \left(\begin{array}{c} \text{cube} \\ D^{-1} \\ \text{cube} \\ F^{-1} \end{array} \right)$$

Using 3-cycles

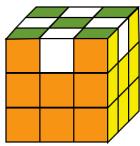


$$\text{left: } \left(\begin{array}{c} \text{cube} \\ L \\ \text{cube} \\ D \end{array} \right) \left(\text{Edge 3-cycle} \right) \left(\begin{array}{c} \text{cube} \\ D^{-1} \\ \text{cube} \\ L^{-1} \end{array} \right)$$

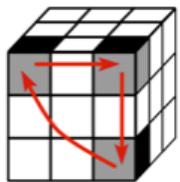
$$\text{right: } \left(\begin{array}{c} \text{cube} \\ R^{-1} \\ \text{cube} \\ D^{-1} \end{array} \right) \left(\text{Edge 3-cycle} \right) \left(\begin{array}{c} \text{cube} \\ D \\ \text{cube} \\ R \end{array} \right)$$

Last Layer : Flip cube over so last layer is now on top.

Step 4: Solve corners in the last layer.

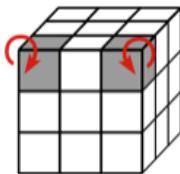


4(a) permute corners



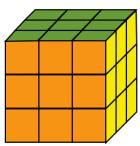
$$\left(\begin{matrix} L & D^{-1} & L^{-1} \\ U & L & D \\ U^{-1} & L^{-1} & U^{-1} \end{matrix} \right)$$

4(b) orient

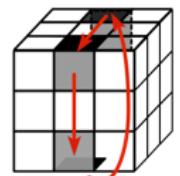


$$\left(\begin{matrix} L & D & x2 & L^{-1} & F^{-1} & D & x2 & F \\ F^{-1} & D & x2 & F & L & D & x2 & L^{-1} \\ U & & & & & & & U^{-1} \end{matrix} \right)$$

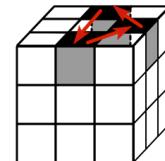
Step 5: Solve edges in the last layer



5(a) permute

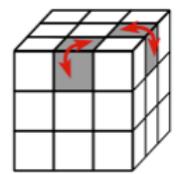


$$\left(\begin{matrix} M & U & M^{-1} & U \\ M^{-1} & D & M & D^{-1} \\ M & D & M^{-1} & D^{-1} \\ M & U & M^{-1} & U^{-1} \end{matrix} \right)$$



$$R^{-1}D^{-1}[M, u^2]DR$$

5(b) orient



$$\left(\begin{matrix} M^{-1} & D & M & D^{-1} & M^{-1} & D & x2 & M \\ M^{-1} & D^{-1} & M & D & M^{-1} & D^{-1} & M & U \\ M & D & M^{-1} & D^{-1} & M & D & M^{-1} & U^{-1} \end{matrix} \right)$$

Summary guide available on course website:

<http://www.sfu.ca/~jtmulhol/math302/downloads/rc-5step-soln.pdf>

Example: Show the steps involved in solving the last layer.

