Chapter 21 - Subgroups of RC_3

What is the group operation on the 4-tuples?

Let X, Y be two elements in RC_3:

$$X = (\rho, \sigma, \vec{v}, \vec{w}) \quad , \quad Y = (\rho^*, \sigma^*, \vec{v}^*, \vec{w}^*).$$

If we compose moves X & Y then the position vector of XY can be obtained as follows:

- corner cubie i moves to
- edge cubie i moves to
- the label on the i^{th} corner cubie, in the primary facet of cubicle $(\rho\rho^*)(i)$, is
- the label on the i^{th} edge cobie, in the primary facet of cubicle $(\sigma\sigma^*)(i)$, is

Therefore,

$$XY = (\rho\rho^*, \sigma\sigma^*, \quad , \quad)$$

where
and

The centre of RC_3:

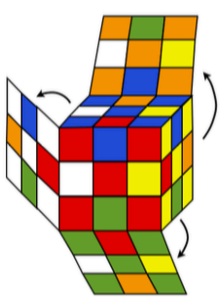
Recall the centre of a group G is:

$$Z(G) = \{ a \in G \mid ag = ga \text{ for all } g \in G \}$$

Let's find the centre of RC_3.

Theorem 21.3.2 The centre of RC_3 consists of two elements: the identity ε and the superflip X_{SF}. The superflip, is the configuration in which every cubie is in its home location but all the edge cubies are flipped (see Figure 21.3).

$$Z(RC_3) = \{\varepsilon, X_{SF}\}.$$



$\rho =$
$\sigma =$
$\vec{v} =$
$\vec{w} =$

Figure 21.3: The superflip configuration of Rubik's cube: X_{SF}.

Proof:

The *Pop Mech* Rubik's Cube Proof

Now, if you're thinking inquisitively, you could desire proof for some of the claims in the last paragraphs. Is there some deeper math that can prove "there's no algorithm that flips one edge cubie in place without moving any other cubie"? You bet. Here's how that mathematical proof roughly goes:

> When a face of the cube is turned, four edge cubies get moved. Consider, for instance, an algorithm of 10 moves. For each cubie, follow it through the algorithm, and count how many times it gets moved, and call that its cubie-moves count. Add up those numbers for every edge cubie, and the total must come to 40 cubie-moves, since each of the 10 moves adds four to the total.

> In general, any algorithm's total number of cubie-moves for the edge cubies must be a multiple of 4. Now for a critical pair of facts: If an edge cubie is moved an even number of times and returned back to the same slot, it will have the same orientation. Conversely, if an edge cubie is moved an odd number of times and put back in the same slot, it will be flipped.