Math 301: End of Course Learning Outcomes

End of course learning outcomes include **understanding** of the material as well as the **mechanics**.

If there is a final exam it will cover material from **Lecture 1 to Lecture 22**.
I will not ask any questions about the Hungarian Rings puzzle.

At the end of the course, before a final exam if there is one, you should:

- ☐ Re-read all the lecture notes again beginning with Lecture 1. You'll really see how much you learned when you look back at previous material and see how the tools we developed later in the term give you new insight into those old problems.
- ☐ Make sure you have a look at the solutions to *all* homework assignment questions, and thoroughly understand how to do these questions.
- ☐ Get a lot more practice solving problems by working through other exercises from the lecture notes booklet.

1 Mechanics

Be able to perform routine mechanical calculations of the following types.

- ☐ Permutations:
  - ☐ representations: disjoint cycle form, array form, arrow form, cycle arrow form; and converting between representations
  - ☐ calculate: composition (multiplication), inverses, order, parity
  - ☐ size of symmetric group S_n, size of alternating group A_n
  - ☐ decompose a permutation into a product of 2-cycles
  - ☐ decompose an even permutation into a product of 3-cycles
  - ☐ use the Orbit-Stabilizer theorem to determine the order of a group
- ☐ Other Groups:
  - ☐ calculating in $\mathbb{Z}_n$
  - ☐ calculating in D_n
- ☐ Puzzles:
  - ☐ represent a puzzle position as a permutation (Definition 5.1)
  - ☐ represent a puzzle move as a permutation (Definition 5.2)
  - ☐ 15-puzzle:
    - ☐ determining solvability by applying the solvability criteria
    - ☐ solve the 15-puzzle
  - ☐ Oval-Track puzzle:
    - ☐ determining solvability by applying the solvability criteria
    - ☐ using the fundamental 3-cycle σ_3, and fundamental 2-cycle σ_2 describe a strategy to solve the puzzle for a given configuration
  - ☐ Rubik's Cube:
    - ☐ solve Rubik's Cube
    - ☐ using the fundamental moves C1, C1', C2, C3, C3', E1, E1', E2, E3' describe a strategy to solve the puzzle for a given configuration
    - ☐ determine the position vector from a given configuration
    - ☐ draw a configuration of cubies from a given position vector
    - ☐ determine the solvability of a configuration using the Fundamental Theorem of Cubology
    - ☐ determining when two assembled cubes are equivalent
    - ☐ determining the quickest way to fix an unsolvable cube (similar to Assignment 9, exercise 2)

2 Definitions

Asking for the statement of a definition of a term on an exam is meant to be easy points. Don't loose these easy points, know your definitions!

Be able to provide the definitions of the following terms:

- ☐ sets, functions, and relations
  - ☐ function, injective (one-to-one), surjection (onto), bijection
  - ☐ partition of a set
  - ☐ relation on a set
    - ☐ reflexive, symmetric, transitive
    - ☐ equivalence relation
      - ☐ equivalence class
      - ☐ equivalence class representative
      - ☐ set of equivalence class representatives
- ☐ permutations:
  - ☐ permutation of a set X
  - ☐ parity of a permutation (Definition 7.1), sign of a permutation (Definition 7.2)
  - ☐ the symmetric group S_n, the alternating group A_n
  - ☐ fixed set of a permutation ($\mathrm{fix}(\alpha)$), moved set of a permutation ($\mathrm{mov}(\alpha)$)
  - ☐ orbit of an element ($\mathrm{orb_G(x)}$), stabilizer of an element ($\mathrm{stab_G(x)}$)
- ☐ group
  - ☐ subgroup
  - ☐ subgroup generated by $g_1, \ldots g_k$
  - ☐ **order** of a group
  - ☐ **order** of an element of a group
  - ☐ Cayley (multiplication) table for a group (Section 10.1.1)
  - ☐ cyclic group (Lecture 10)
  - ☐ abelian group (last paragraph of Section 10.2)
  - ☐ commutator (Definition 13.1)
  - ☐ conjugate (Definition 14.1, 14.2)
  - ☐ cosets (Lecture 18)
  - ☐ examples:
    - ☐ group of integers modulo n: $\mathbb{Z}_n$
    - ☐ dihedral group of a regular n-gon: D_n
- ☐ Rubik's cube: cubies, cubicles, stickers and facets; home location, home orientation; orientation markings; position vector (Definition 20.1); illegal cube group RC_3^*, legal cube group RC_3.

3 Theorems

Know how to state, and use the following theorems.
(Like definitions, know the statements of theorems for easy points.)

- ☐ Relations and Partitions: Lemma 17.1 and Theorem 17.1
- ☐ Permutations:
  - ☐ parity theorem (Theorem 7.1)
  - ☐ decompositions into 2-cycles and 3-cycles (Theorems 6.1 and 8.2)
  - ☐ orbit-stabilizer theorem

- ☐ groups in general
  - ☐ Lagrange's Theorem (Theorem 11.3, restated in 18.1)
  - ☐ Cyclic group theorems (Theorems 11.5 -11.8)
  - ☐ conjugation preserves cycle structure (Lemma 14.1)
  - ☐ properties of cosets (Lemma 18.2)
- ☐ Puzzle specific theorems:
  - ☐ Multiplying Puzzle Moves (Theorem 5.1)
  - ☐ solvability criteria for 15-puzzle (Theorems 9.1, 9.2)
  - ☐ solvability criteria for Oval Track puzzle (Theorem 15.1)
  - ☐ Fundamental Theorem of Cubology (Theorem 20.1)

4 Know how to explain . . .

- ☐ . . . why the product of two even permutations is even, the product of two odd permutations is even, and the product of an odd and an even permutation is odd..
- ☐ . . . to produce an odd permutation of the Oval Track puzzle a move sequence must put every disk in the turntable at least one (Section 15.1.1).
- ☐ . . . how changing the number of disks on the Oval Track puzzle affects the solvability of the puzzle (15.1.4).
- ☐ . . . the connection between an equivalence relation on a set and a partition of a set.
- ☐ . . . the connection between commutators (specifically Equation (2) in Section 13.2) and creating useful moves on a puzzle.
- ☐ . . . the connection between conjugation and modifying existing puzzle moves.

5 Provided on Exam

If there is a final exam these are the things I will provide you with in the exam room.

1) Oval Track puzzle: fundamental 2-cycle: $\sigma_2 = (TR^{-1})^{17} = (1, 3)$ and fundamental 3-cycle: $\sigma_3 = [R^{-3}, T]^2 = (1, 7, 4)$.
2) Rubik's Cube: diagrams showing cubie/cubicle/facet lablelings, along with facet orientation markings and sticker orientation labels (as shown in Figures 20.2, 20.3)

You can bring your Rubik's Cube, 15-puzzle, and swap puzzle to any exam.