Appendix: Model and Experiments

1. Model

A. Model solution under rational expectations

Denoting $\pi_t^e = E_t \pi_{t+1}$, $x_t^e = E_t x_{t+1}$, we can write the rational expectations solution of the equilibrium system (1)–(3) as

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} A_{\pi\pi} & A_{\pi x} \\ A_{x\pi} & A_{xx} \end{bmatrix} \begin{bmatrix} \pi_{t-1}^e \\ x_{t-1}^e \end{bmatrix} + \begin{bmatrix} B_\pi \\ B_x \end{bmatrix} r_t^n$$

The equilibrium system of equations can be written as

$$E_t \left[A_1 \begin{bmatrix} \pi_t^e \\ x_t^e \\ \pi_{t+1} \\ x_{t+1} \end{bmatrix} + A_2 \begin{bmatrix} \pi_{t-1}^e \\ x_{t-1}^e \\ \pi_t \\ x_t \end{bmatrix} + shock\ terms \right] = 0$$

where

$$A_1 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & \sigma^{-1} & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & \kappa \\ -\sigma^{-1}\phi_\pi & -\sigma^{-1}\phi_x & 0 & -1 \end{bmatrix}$$

Following the method in Blanchard and Khan (1980), we can construct generalized eigenvalues of $-A_1^{-1}A_2$, which will give us $A_1^{-1}A_2 = -V\Lambda V^{-1}$, where V and Λ are matrixes of eigenvectors and eigenvalues, respectively. Eigenvalues are found by solving the generalized eigenvalue problem:

$$\det\left(A_1^{-1}A_2 + \lambda I\right) = 0$$

This gives

$$\frac{\lambda^2}{\beta}\left(\beta\lambda^2-\left(1+\beta+\kappa\sigma^{-1}+\beta\sigma^{-1}\phi_x\right)\lambda+1+\kappa\sigma^{-1}\phi_\pi+\sigma^{-1}\phi_x\right)=0$$

So there are four generalized eigenvalues: $\lambda_{1,2}=0$, $\lambda_{3,4}=\frac{\left(1+\beta+\kappa\sigma^{-1}+\beta\sigma^{-1}\phi_x\right)}{2\beta}\pm\frac{\sqrt{(1-\beta+\kappa\sigma^{-1}+\beta\sigma^{-1}\phi_x)^2-4\beta(\kappa\sigma^{-1}(\phi_\pi-1)+\sigma^{-1}\phi_x(1-\beta))}}{2\beta}$. The infinite eigenvalues correspond to $A_1^{-1}A_2\mathbf{v}=0$; i.e., they correspond to eigenvectors $\left[1,0,-\kappa\sigma^{-1}\phi_\pi,-\sigma^{-1}\phi_\pi\right]'$ and $\left[0,1,-\kappa\sigma^{-1}\phi_x,-\sigma^{-1}\phi_x\right]'$. The equilibrium is locally determinate if and only if $|\lambda_{2,3}|>1$, or equivalently if $\phi_\pi+\phi_x\frac{1-\beta}{\kappa}>1$. This is also known as the Taylor principle.[1] The fact that the stable eigenvalue that is equal to zero implies that E_tx_{t+1} and $E_t\pi_{t+1}$ does not depend on the endogenous state variables π_{t-1}^e, x_{t-1}^e, but only on the exogenous state variable. Hence the solution is

$$\begin{bmatrix}\pi_t\\ x_t\end{bmatrix}=-\sigma^{-1}\begin{bmatrix}\kappa\phi_\pi & \kappa\phi_x\\ \phi_\pi & \phi_x\end{bmatrix}\begin{bmatrix}\pi_{t-1}^e\\ x_{t-1}^e\end{bmatrix}+\begin{bmatrix}B_\pi\\ B_x\end{bmatrix}r_t^n$$

this implies that

$$E_t\begin{bmatrix}\pi_{t+1}\\ x_{t+1}\end{bmatrix}=\begin{bmatrix}\Phi_\pi\\ \Phi_x\end{bmatrix}\rho_r r_t^n$$

where

$$\begin{aligned}\begin{bmatrix}\Phi_\pi\\ \Phi_x\end{bmatrix} &= \Delta^{-1}\begin{bmatrix}\left(\sigma^2+\phi_x\sigma\right)B_\pi-\sigma\phi_xB_x\\ -\sigma\phi_\pi B_\pi+\sigma\left(\sigma+\kappa\phi_\pi\right)B_x\end{bmatrix}\\ \text{and }\ \Delta &= \sigma^2+\sigma\left(\phi_x+\kappa\phi_\pi\right)+\left(\kappa-1\right)\phi_\pi\phi_x\end{aligned}$$

[1] See Woodford (2003, Chapter 4).

Plug the solution into the system and use the method of undetermined coefficients:

$$
\begin{aligned}
B_x &= \Delta^{-1}\left(-\sigma\phi_\pi B_\pi + \sigma\left(\sigma+\kappa\phi_\pi\right)B_x\right)\rho_r \\
&\quad -\sigma^{-1}\left(-\Delta^{-1}\left(\left(\sigma^2+\phi_x\sigma\right)B_\pi - \sigma\phi_x B_x\right)\rho_r - 1\right) \\
B_\pi &= \kappa B_x + \beta\Delta^{-1}\left(\left(\sigma^2+\phi_x\sigma\right)B_\pi - \sigma\phi_x B_x\right)\rho_r
\end{aligned}
$$

This gives

$$
\begin{bmatrix} B_\pi \\ B_x \end{bmatrix} = \Gamma \begin{bmatrix} \kappa\left(1+\left(\phi_\pi\kappa+\phi_x\right)\sigma^{-1}\right)+\phi_x\sigma^{-1}\left(\kappa\left(\kappa-1\right)\phi_\pi\sigma^{-1}-\beta\rho_r\right) \\ 1+\left(\phi_\pi\kappa+\phi_x\right)\sigma^{-1}-\beta\rho_r+\phi_x\sigma^{-1}\left(\left(\kappa-1\right)\phi_\pi\sigma^{-1}-\beta\rho_r\right) \end{bmatrix}
$$

where $\Gamma = \frac{\sigma^{-1}}{(1-\beta\rho_r)(1-\rho_r)+\sigma^{-1}\kappa(\phi_\pi-\rho_r)+\sigma^{-1}\phi_x((\kappa-1)(\phi_\pi-\rho_r)\sigma^{-1}+1-\beta\rho_r)}$.

The decision rules are

$$
\begin{aligned}
\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} &= -\sigma^{-1}\begin{bmatrix} \kappa\phi_\pi & \kappa\phi_x \\ \phi_\pi & \phi_x \end{bmatrix}\begin{bmatrix} \pi^e_{t-1} \\ x^e_{t-1} \end{bmatrix} \\
&\quad +\Gamma\begin{bmatrix} \kappa\left(1+\left(\phi_\pi\kappa+\phi_x\right)\sigma^{-1}\right)+\phi_x\sigma^{-1}\left(\kappa\left(\kappa-1\right)\phi_\pi\sigma^{-1}-\beta\rho_r\right) \\ 1+\left(\phi_\pi\kappa+\phi_x\right)\sigma^{-1}-\beta\rho_r+\phi_x\sigma^{-1}\left(\left(\kappa-1\right)\phi_\pi\sigma^{-1}-\beta\rho_r\right) \end{bmatrix} r^n_t
\end{aligned}
$$

which also implies the following forecasting rules:

$$
E_t\begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} \Phi_\pi \\ \Phi_x \end{bmatrix}\rho_r r^n_t
$$

B. Model solution under non-rational expectations

In the model, agents make decisions according to the decision rules as functions of the past history of exogenous events (shocks). These decision rules characterize optimal choices regarding consumption, savings and prices, given how agents form expectations with respect to future economic outcomes. Under rational expectations, these expectations are a statistical mean over all possible outcomes implied by decisions conditional on every future history of events. This implies that agents always behave in a way consistent with their decision rules, and that this behaviour is based on the most likely turn of random events. Therefore, deviation from rational expectations implies that agents do not behave consistently or that they make systematic errors in forecasting random events.

Although rational expectations represent a useful benchmark for how agents form their expectations, we need to understand the implications of assuming alternative expectations formation functions for the design of monetary policy. In general, the expected values of the output gap and inflation, $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$, are linear functions of the state history, $\{r_s^n\}_{s=0}^t$. As we showed in section A of this appendix, under rational expectations these functions are only functions of r_t^n. These functions imply that agents' forecast errors are zero, on average. To understand the effect of alternative expectations formation functions on outcomes (and the expectations channel), assume that period-t forecast errors co-vary with the state in periods t and $t-1$; i.e., that

$$E_t \left(E_t^* \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} - \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} \right) = \sigma^{-1} \rho_r \left(\begin{bmatrix} \kappa L_{0\pi} \\ L_{0x} \end{bmatrix} r_t^n + \begin{bmatrix} \kappa L_{1\pi} \\ L_{1x} \end{bmatrix} \rho_r r_{t-1}^n \right)$$

where $L_{0\pi}$, L_{0x}, $L_{1\pi}$, L_{1x} are real numbers representing the elasticity of forecast errors on inflation and the output gap with respect to shock realizations in periods t and $t-1$. The above specification of the forecast errors implies that agents' expectations are inconsistent with rational expectations during the first two quarters after the shock and are, on average, zero afterwards. The case with

$L_{0\pi} = L_{0x} = L_{1\pi} = L_{1x} = 0$ corresponds to rational expectations. Hence, the above specification allows us to study deviations from rational expectations that occur only with respect to current or last-period state realizations. We consider another specification in which forecast errors correlate with the longer history of shocks afterwards.

To solve our equilibrium system (1)-(3) under a given specification for the expectations (see section 2 of the main text), expectations must be of the form

$$E_t^* \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} \theta_\pi \\ \theta_x \end{bmatrix} r_t^n + \begin{bmatrix} \eta_\pi \\ \eta_x \end{bmatrix} \rho_r r_{t-1}^n$$

To find the unknown coefficients θ_π, θ_x, η_π, η_x, plug this equation into (1)–(3) to get the outcomes as

$$\begin{aligned}
\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} &= \left(\begin{bmatrix} -\kappa\sigma^{-1}\phi_\pi + \kappa\sigma^{-1}\rho_r + \beta\rho_r & -\kappa\sigma^{-1}\phi_x + \kappa\rho_r \\ -\sigma^{-1}\phi_\pi + \sigma^{-1}\rho_r & -\sigma^{-1}\phi_x + \rho_r \end{bmatrix} \begin{bmatrix} \theta_\pi \\ \theta_x \end{bmatrix} \right. \\
&\quad \left. + \begin{bmatrix} \beta\rho_r + \kappa\sigma^{-1}\rho_r & \kappa\rho_r \\ \sigma^{-1}\rho_r & \rho_r \end{bmatrix} \begin{bmatrix} \eta_\pi \\ \eta_x \end{bmatrix} + \sigma^{-1}\rho_r \begin{bmatrix} \kappa \\ 1 \end{bmatrix} \right) r_{t-1}^n \\
&\quad + \begin{bmatrix} -\kappa\sigma^{-1}\phi_\pi\rho_r & -\kappa\sigma^{-1}\phi_x\rho_r \\ -\sigma^{-1}\phi_\pi\rho_r & -\sigma^{-1}\phi_x\rho_r \end{bmatrix} \begin{bmatrix} \eta_\pi \\ \eta_x \end{bmatrix} r_{t-2}^n \\
&\quad + \left(\begin{bmatrix} \kappa\sigma^{-1} + \beta & \kappa \\ \sigma^{-1} & 1 \end{bmatrix} \begin{bmatrix} \theta_\pi \\ \theta_x \end{bmatrix} + \sigma^{-1} \begin{bmatrix} \kappa \\ 1 \end{bmatrix} \right) \varepsilon_t
\end{aligned}$$

Use the method of undetermined coefficients to find θ_π, θ_x, η_π, η_x. Denoting $h_0 = \frac{\rho_r}{1+\kappa\sigma^{-1}\phi_\pi+\kappa\sigma^{-1}\phi_x(1+(\kappa-1)\sigma^{-1}\phi_\pi)}$, we get

$$\begin{bmatrix} \eta_\pi \\ \eta_x \end{bmatrix} = \Gamma_{\eta 1} \begin{bmatrix} \kappa L_{1\pi} \\ L_{1x} \end{bmatrix}$$

and

$$\begin{bmatrix} \theta_\pi \\ \\ \theta_x \end{bmatrix} = \Gamma_{\theta 0} \begin{bmatrix} \kappa\left(1+L_{0\pi}\right) \\ \\ 1+L_{0x} \end{bmatrix} + \Gamma_{\theta 1} \begin{bmatrix} \kappa L_{1\pi} \\ \\ L_{1x} \end{bmatrix}$$

where

$$\begin{aligned}
\Gamma_{\eta 1} &= \sigma^{-1} h_0 \begin{bmatrix} 1+\kappa\phi_x\sigma^{-1} & -\kappa\sigma^{-1}\phi_x \\ \\ -\sigma^{-1}\phi_\pi & 1+\kappa\sigma^{-1}\phi_\pi \end{bmatrix} \\
\Gamma_{\theta 0} &= \Gamma\rho_r \begin{bmatrix} 1-\rho_r+\phi_x\sigma^{-1} & \kappa\left(\rho_r-\phi_x\sigma^{-1}\right) \\ \\ -\sigma^{-1}\left(\phi_\pi-\rho_r\right) & 1+\kappa\sigma^{-1}\left(\phi_\pi-\rho_r\right)-\beta\rho_r \end{bmatrix} \\
\Gamma_{\theta 1} &= \Gamma h_0 \begin{bmatrix} 1-\rho_r+\phi_x\sigma^{-1} & \kappa\left(\rho_r-\phi_x\sigma^{-1}\right) \\ \\ -\sigma^{-1}\left(\phi_\pi-\rho_r\right) & 1+\kappa\sigma^{-1}\left(\phi_\pi-\rho_r\right)-\beta\rho_r \end{bmatrix} \\
&\quad \times \begin{bmatrix} \left(\beta\rho_r+\frac{\kappa}{\sigma}\rho_r\right)\left(\frac{\kappa}{\sigma}\phi_x+1\right)-\frac{\kappa}{\sigma}\phi_\pi\rho_r & \kappa\rho_r\left(\frac{\kappa}{\sigma}\phi_\pi+1\right)-\frac{\kappa}{\sigma}\phi_x\left(\beta\rho_r+\frac{\kappa}{\sigma}\rho_r\right) \\ \\ \frac{1}{\sigma}\rho_r\left(\frac{\kappa}{\sigma}\phi_x+1\right)-\frac{1}{\sigma}\phi_\pi\rho_r & \rho_r\left(\frac{\kappa}{\sigma}\phi_\pi+1\right)-\frac{\kappa}{\sigma^2}\rho_r\phi_x \end{bmatrix}
\end{aligned}$$

Sensitive and static expectations

We consider two cases of forecast-error specification: one in which forecast errors are positively correlated with recent state history and the other with negative correlation. To build intuition for such non-rational expectations, we note that the specification of expectations implies that $E_t^* r_{t+1}^n \neq E_t r_{t+1}^n$; i.e., agents make consistent errors in correctly forecasting the future realization of the state. Moreover, we assume that agents' expectations $E_t^* \pi_{t+1}$ and $E_t^* x_{t+1}$ may not be consistent with a single underlying stochastic process for r_t^n, or, in other words, that their inflation and output-gap expectations are based on different perceptions for the r_t^n process. To emphasize this feature, will use use notation $\overrightarrow{E_t^* r_{t+1}^n}$ for this 2-vector. We obtain the expression for $\overrightarrow{E_t^* r_{t+1}^n}$ by plugging the solution from the previous section into the forecast specification and assuming for concreteness that $L_{1\pi} = L_{1x} = 0$:

$$\left(\sigma^{-1} \begin{bmatrix} \kappa F_\pi \\ F_x \end{bmatrix} + \sigma^{-1} \begin{bmatrix} \kappa \\ 1 \end{bmatrix} \right) \left(E_t^* r_{t+1}^n - \rho_r r_t^n \right) = \sigma^{-1} \rho_r \begin{bmatrix} \kappa L_{0\pi} \\ L_{0x} \end{bmatrix} r_t^n$$

where

$$\begin{bmatrix} F_\pi \\ F_x \end{bmatrix} = \sigma \Gamma \rho_r \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \times \begin{bmatrix} 1 + L_{0\pi} \\ 1 + L_{0x} \end{bmatrix}$$

and where

$$\begin{aligned}
a_1 &= \left(\beta + \frac{\kappa}{\sigma} \right) \left(\frac{1}{\sigma} \phi_x - \rho_r + 1 \right) - \frac{\kappa}{\sigma} \left(\phi_\pi - \rho_r \right) \\
a_2 &= \left(\frac{\kappa}{\sigma} \left(\phi_\pi - \rho_r \right) - \beta \rho_r + 1 \right) + \left(\beta + \frac{\kappa}{\sigma} \right) \left(\rho_r - \frac{1}{\sigma} \phi_x \right) \\
a_3 &= \kappa \left(\frac{1}{\sigma} \left(\frac{1}{\sigma} \phi_x - \rho_r + 1 \right) - \frac{1}{\sigma} \left(\phi_\pi - \rho_r \right) \right) \\
a_4 &= \frac{\kappa}{\sigma} \left(\phi_\pi - \rho_r \right) - \beta \rho_r + \frac{\kappa}{\sigma} \left(\rho_r - \frac{1}{\sigma} \phi_x \right) + 1
\end{aligned}$$

The above equation implies that the perceived persistence of the fundamental shock, $\frac{cov\left(\overrightarrow{E_t^* r_{t+1}^n}, r_t^n\right)}{var(r_t^n)}$, is not equal to ρ_r, and moreover, that the shock's persistence depends on whether it is inferred from inflation or output-gap dynamics. Let us denote the persistence of the shock consistent with inflation (output-gap) dynamics by $\rho_r^{*\pi}$ (ρ_r^{*x}); i.e.,

$$\overrightarrow{E_t^* r_{t+1}^n} = \begin{bmatrix} \rho_r^{*\pi} \\ \rho_r^{*\pi} \end{bmatrix} r_t^n$$

Then we obtain that

$$\begin{aligned}\rho_r^{*\pi} &= \rho_r\left(1+\frac{L_{0\pi}}{1+F_\pi}\right)\\ \rho_r^{*x} &= \rho_r\left(1+\frac{L_{0x}}{1+F_x}\right)\end{aligned}$$

Under rational expectations, $L_{0\pi} = L_{0x} = 0$, agents correctly infer that

$$\rho_r^{*\pi} = \rho_r^{*x} = \rho_r$$

Alternatively, if $L_{0\pi} > 0$ and $L_{0x} > 0$, then

$$\begin{aligned}\rho_r^{*\pi} &> \rho_r\\ \rho_r^{*x} &> \rho_r\end{aligned}$$

i.e., forecast errors are equivalent to perceiving the shock as more persistent than it is. This case implies that period-t forecasts of inflation and the output gap (relative to their rational forecasts) are positively correlated with period-t shock realizations. This is equivalent to saying that period-$(t+1)$ forecast errors are negatively correlated with period-t shocks. For example, if in period t there is a positive shock to the real interest rate, $r_t^n > 0$, then agents' forecasts tend to be more elastic with respect to rational forecasts. For this reason, we term such expectations formation as *sensitive expectations*.

If, instead, the deviation from the rational expectations goes in the opposite direction (i.e., if $L_{0\pi} < 0$ and $L_{0x} < 0$), then the perceived shock persistence is lower than it is:

$$\begin{aligned} \rho_r^{*\pi} &< \rho_r \\ \rho_r^{*x} &< \rho_r \end{aligned}$$

In this case, period-$(t+1)$ forecast errors are *positively* correlated with period-t shocks. For example, if in period t there is a positive shock to the real interest rate, $r_t^n > 0$, then agents' forecasts tend to be *less* elastic with respect to rational forecasts. We therefore term these expectations *static expectations*. For a particular case with $L_{0\pi} = L_{0x} = -1$, agents perceive that the fundamental shock is i.i.d., $\rho_r^{*\pi} = \rho_r^{*x} = 0$, so that $E_t^* x_{t+1} = E_t^* \pi_{t+1} = 0$.

Adaptive expectations

So far, we have studied deviations from rational expectations that implied that agents' forecast errors do no persist for a long period of time. In particular, we have considered the case where those errors were consistent with rational expectations, but for only the first two periods after the shock. To investigate the implications of forecast errors that persist for a long time, we turn to another specification of non-rational expectations. We draw on the experimental literature and assume that the expected values of inflation and the output gap are functions of past realizations of inflation and the output gap. Specifically, we assume that

$$E_t^* \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} = (1-\omega) E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} + \omega \begin{bmatrix} \pi_{t-l} \\ x_{t-l} \end{bmatrix}, \quad \omega \in [0,1]$$

i.e., that period-t expected values of inflation and the output gap in period-$(t+1)$ are weighted averages of statistical expectations of inflation and the output gap in period-$(t+1)$ (with weight

$1-\omega$) and their realizations in period $t-l$. The implied expected forecast errors are

$$E_t\left(E_t^*\begin{bmatrix}\pi_{t+1}\\ x_{t+1}\end{bmatrix}-\begin{bmatrix}\pi_{t+1}\\ x_{t+1}\end{bmatrix}\right)=-\omega\left(E_t\begin{bmatrix}\pi_{t+1}\\ x_{t+1}\end{bmatrix}-\begin{bmatrix}\pi_{t-l}\\ x_{t-l}\end{bmatrix}\right)$$

Therefore, in period t agents use period $t-l$ realization of inflation and the output gap to form expectations of period-$(t+1)$ inflation and the output gap. If realized inflation or the output gap in period $t-l$ is high (low), then agents' forecasts of inflation and the output gap tend to be higher (lower) than would be implied under rational expectations. We therefore term such expectations *adaptive expectations.*

One important implication of adaptive expectations is that, unlike in the case of static or sensitive expectations, agents' forecast errors persist forever. To see this, note that the solution to (1)–(3) under the above condition on expectations takes the AR form

$$\begin{bmatrix}\pi_t\\ x_t\end{bmatrix}=\begin{bmatrix}A_\pi\\ A_x\end{bmatrix}r_{t-1}^n+\begin{bmatrix}B_\pi\\ B_x\end{bmatrix}r_t^n+\begin{bmatrix}C_\pi\\ C_x\end{bmatrix}\begin{bmatrix}\pi_{t-1}\\ x_{t-1}\end{bmatrix},\qquad \text{if } l=0$$

and

$$\begin{bmatrix}\pi_t\\ x_t\end{bmatrix}=\begin{bmatrix}A_\pi\\ A_x\end{bmatrix}\begin{bmatrix}\pi_{t-l-1}\\ x_{t-l-1}\end{bmatrix}+\begin{bmatrix}B_\pi\\ B_x\end{bmatrix}r_t^n+\begin{bmatrix}C_\pi\\ C_x\end{bmatrix}\begin{bmatrix}\pi_{t-l}\\ x_{t-l}\end{bmatrix},\qquad \text{if } l=1,2,...$$

Unlike sensitive expectations, for which such effects last a finite number of periods, period-t shock realization has long-lasting effects on agents' forecast errors.

Figure A.1 shows impulse responses in the model with rational expectations.

Figure A.2 provides the fraction of inflation variance decreased via expectations in the model with rational expectations, for a range of key parameter values.

Figure A.3 gives that fraction for alternative expectations formations.

Figure A.4 compares forecasts and forecast errors estimated for the experiment with those estimated for the model with sensitive, adaptive(2) and adaptive(3) forms of expectations.

C. Calibration of model parameters

All data are at a quarterly frequency, spanning the inflation targeting period in Canada, from 1992Q1 to 2012Q2. The output gap and all trends are calculated by the Bank of Canada.[2] Inflation is based on Statistics Canada's v41690914 series: "Consumer price index (CPI) seasonally adjusted 2005 basket - Canada; All-items." The nominal interest rate is based on the Bank of Canada's v39078 series "Bank rate."

The standard deviation of inflation is 0.44 per cent. Standard deviation of the output gap is 1.95 per cent, or 4.4 times the standard deviation of inflation. The persistence of the output gap in the data, 0.79, is much higher than the inflation persistence, 0.09. Since the model does not include mechanisms to account for differences in the persistence of inflation and the output gap, it predicts virtually the same persistence for the output gap and inflation. We therefore calibrate the model to match the persistence of inflation to 0.4, which is at the midpoint between inflation and output-gap persistence in the data. It is also close to inflation persistence over the longer historical time period, 1973:3-2012:2. In the end, three model parameters (standard deviation and the serial correlation of the r_t^n shock process, σ_r and ρ_r, and the slope of the New Keynesian Phillips curve, κ), are calibrated to match the following three calibration targets: standard deviation and the serial correlation of inflation deviations, 0.44 per cent and 0.4, and the ratio of standard deviations of the output gap and inflation, 4.4. Table A.1 summarizes the calibrated parameters and calibration targets.

[2] Trend values for the output gap, inflation and the interest rate can be provided upon request.

2. Experiments

Boxes A.1 and A.2 provide snapshots of the forecast and history screens.

Boxes A.3 and A.4 contain the texts of non-technical and technical instructions, respectively.

A. Pilot sessions

Prior to conducting our final experiment, we ran seven pilot sessions. These sessions allowed us to refine our instructions and design. We tried a number of variations on the instructions. In some pilots, we provided participants with highly numerical descriptions of the economy (i.e., full calibrations of the system). Subjects complained about the perceived technical nature of the environment, and commented that they were overwhelmed with too much information.

We also conducted pilot sessions that involved minimal instructions similar to thase of Bao et al. (2012) and Pfajfar and Žakelj (2012, 2013), where a qualitative description of the economy was given to subjects during the instruction phase of the experiment. While we explained how different variables interacted and in what direction they would influence one another, there was no discussion of their relative importance. The qualitative description resulted in less confusion during the instructions and experiment, and subjects appeared much more receptive to participating.

In addition, we also made technical instructions available to subjects who would be interested in knowing more details about the set-up. Finally, during the pilots, we found that subjects were far more receptive to the experiment when we walked them through qualitative examples of how each of the different factors affected inflation, output and the nominal interest rate.

In our earlier pilot sessions, participants viewed the last period's outcomes (inflation and output) as well as the implied forecast errors on the main screen. In this case, subjects' forecasts of future outcomes were greatly biased by the past outcomes that they saw on their screens. To avoid priming participants to exhibit such behaviour, in our final design we removed that information from the main screen, and instead allowed subjects to access that information by clicking on the history screen.

B. Communication treatment

In our main experiment, we have assumed no role for the communication of monetary policy. This assumption is consistent with our theoretical framework, in which it is assumed that agents have complete information about the model and, in particular, the way in which monetary policy is set. Specifically, conditional on the realized history of the shock, agents' expectations of inflation and the output gap are consistent with future policy actions implied by the Taylor rule specification in the model.

In this treatment, we test this assumption by adding to our experiment an explicit announcement of the expected path of future nominal interest rates. In period t, subjects will see on the main screen, in addition to the same information as before, conditional expected values of nominal interest rates in the following T_i periods: $E_{t-1}i_{t+1}$, $E_{t-1}i_{t+2}$, ..., $E_{t-1}i_{t+T_i}$. We assume that, to compute the expected path of nominal interest rates after period t, the central bank uses the solution of the model with rational expectations, conditional on history through $t-1$.

In that solution, the interest rate in period t is the following function of the shock:

$$\begin{aligned} i_t &= 1.5\left(0.141 r^n_{t-1}\right) + 0.5\left(0.472 r^n_{t-1}\right) \\ &= 0.448 r^n_{t-1} \end{aligned}$$

This implies that

$$E_{t-1}i_{t+s} = 0.57^s i_t, \quad s = 1, ..., T_i$$

with one-standard-deviation bands given by adding/subtracting from those point values $\sqrt{s}\sigma_r, \quad s = 1, ..., T_i$.

Assumption that subjects make their decisions with a complete understanding of the model (and monetary policy), the communication of future expected monetary policy actions in this treatment *should not* have significant effects on the outcomes, particularly on how effective monetary policy is in stabilizing inflation and output-gap fluctuations. Table A.2 reports experimental results for this treatment.

References

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Table A.1: Model parameterization

A. Calibrated Parameters

σ_r	st dev of $r^n{}_t$ innovations, %	1.13
ρ_r	ser corr of $r^n{}_t$	0.57
κ	slope of NKPC	0.13

B. Targets

	Data	Model
st dev of π_t, %	0.44	0.44
ser corr of π_t	0.40	0.40
$\mathrm{std}(x_t)/\mathrm{std}(\pi_t)$	4.4	4.4

C. Assigned Parameters

	period	1 quarter
β	discount factor	$0.96^{1/4}$
σ	risk aversion	1
ϕ_π	Taylor rule coef, inflation	1.5
ϕ_x	Taylor rule coef, output gap	0.5

Table A.2: Experimental evidence, communication treatment

Treatment	Fraction of conditional variance decreased via expectations channel		std(π_t)	ser.cor.(π_t)	std(x_t)/std(π_t)
	π_t	x_t			
Benchmark					
Model (Rational)	**0.73**	**0.65**	**0.44**	**0.40**	**4.4**
Model (Adaptive 1)	**0.20**	**0.32**	**1.00**	**0.74**	**2.6**
Experiments (Benchmark)					
median	0.51	0.45	0.79	0.56	3.8
min	0.25	0.03	0.54	0.49	3.0
max	0.56	0.56	0.92	0.69	4.1
Experiments (Communication)					
median	0.19	0.10	1.18	0.75	2.9
min	-0.94	-3.47	0.75	0.66	2.5
max	0.59	0.64	2.24	0.82	4.1

Note: Statistics for each treatment in the experiments are computed for five (six) sessions of repetition 2 for the benchmark (communication) treatment.

Figure A.1: Responses to 113 bps impulse to r_t^n shock

Rational expectations

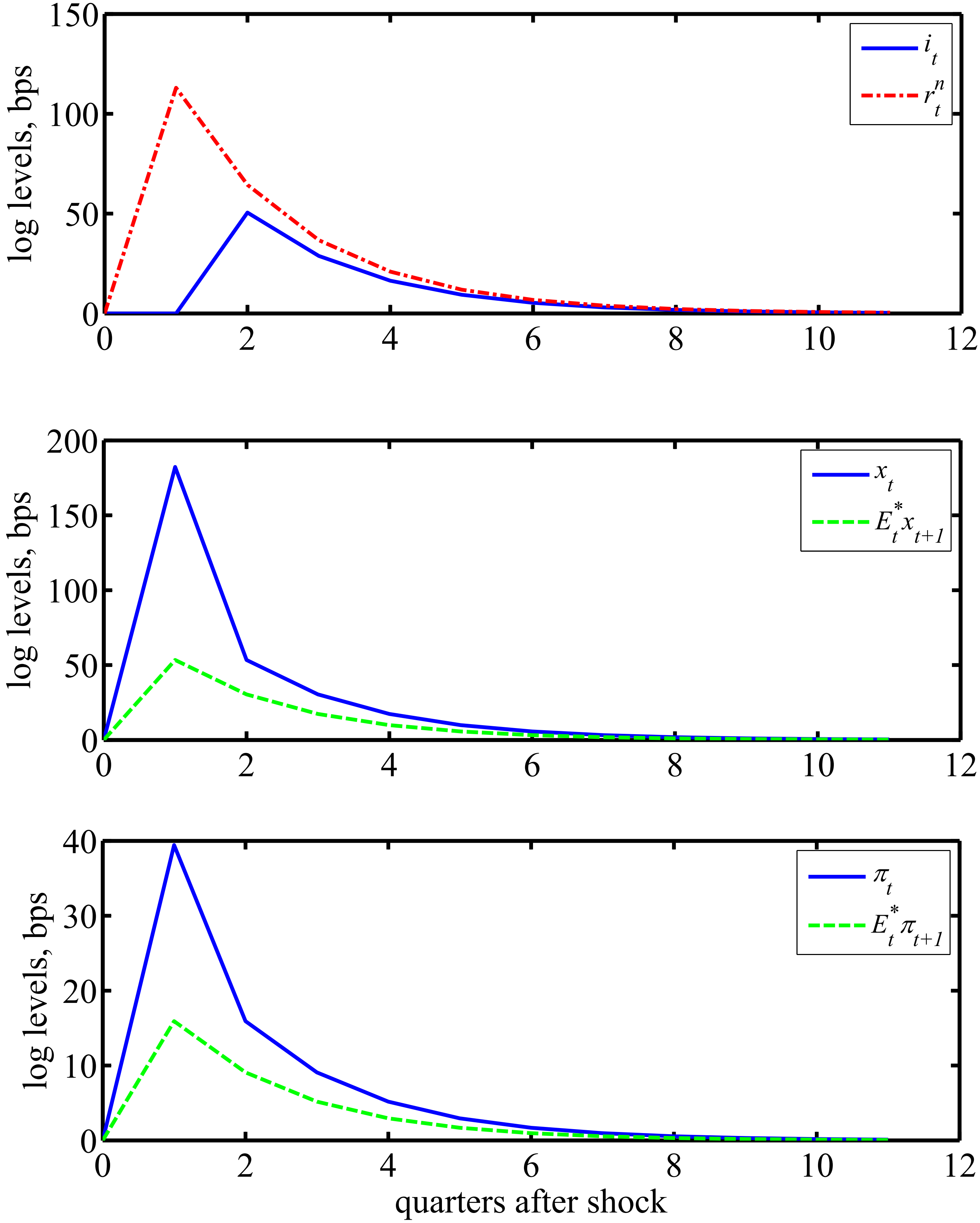


Figure A.2: Fraction inflation variance decreased via expectations,
Rational expectations (o - baseline calibration)

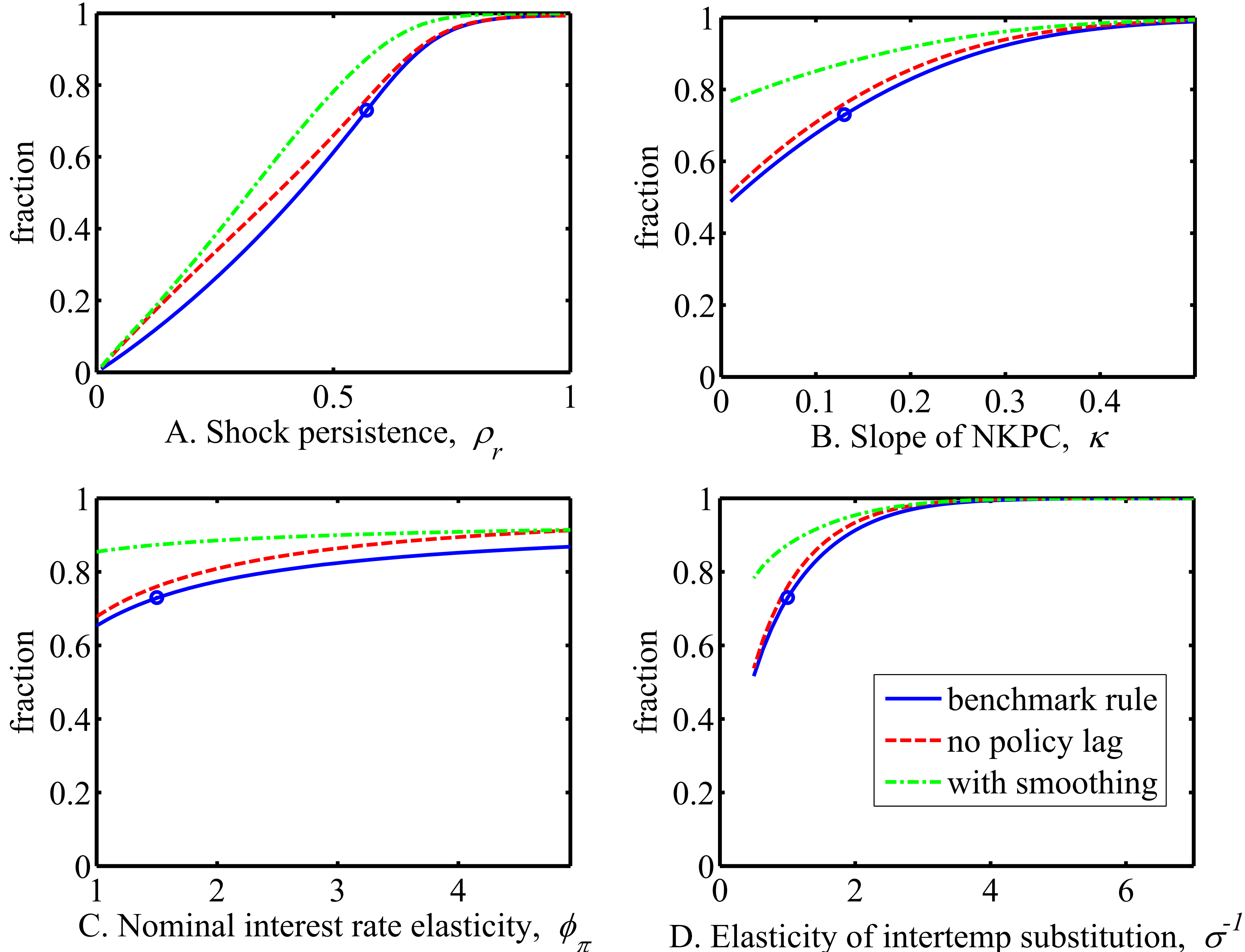


Figure A.3: Fraction of inflation variance decreased via expectations,
(o - baseline calibration)

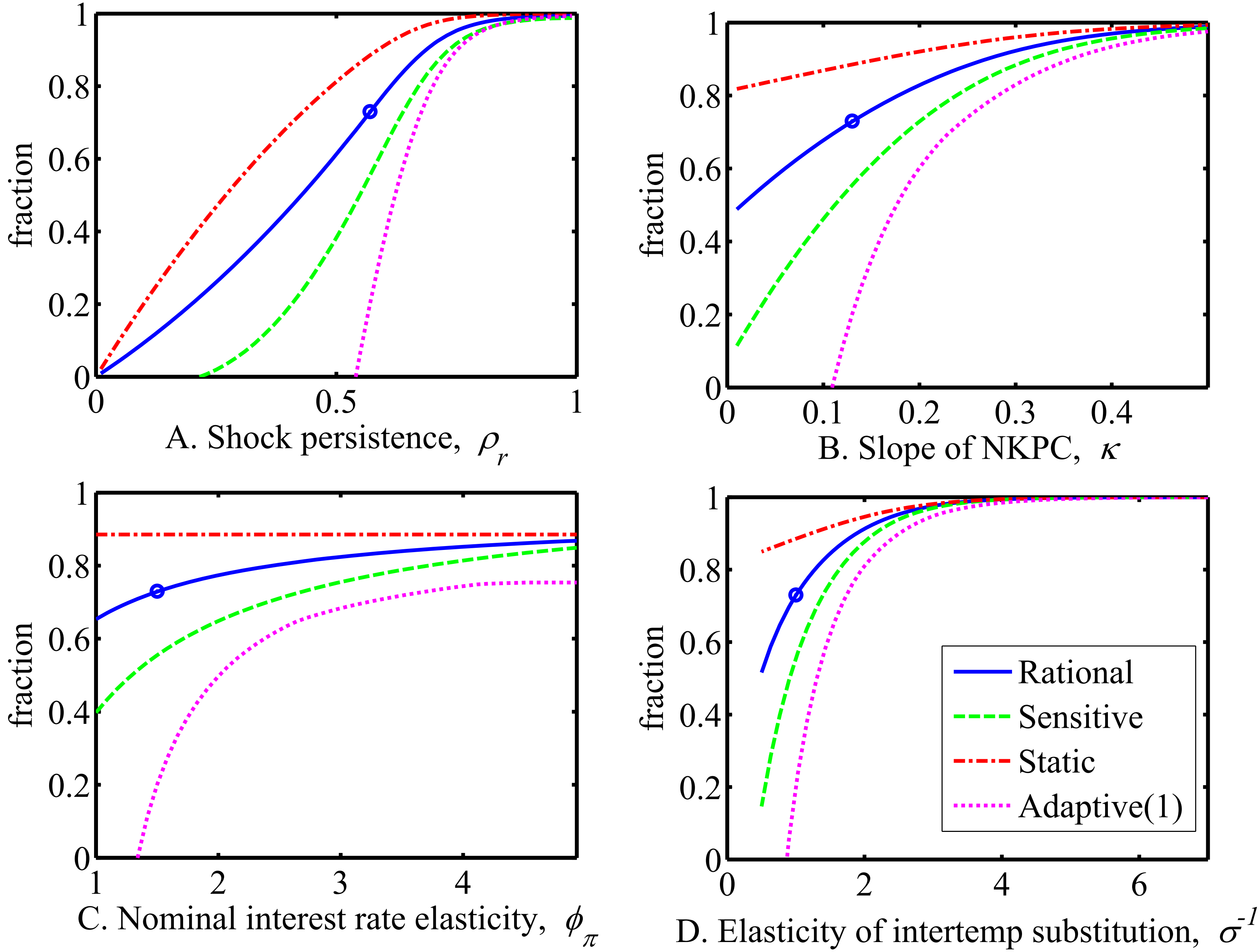


Figure A.4: Responses to 113 bps impulse to natural rate of interest, r_t^n

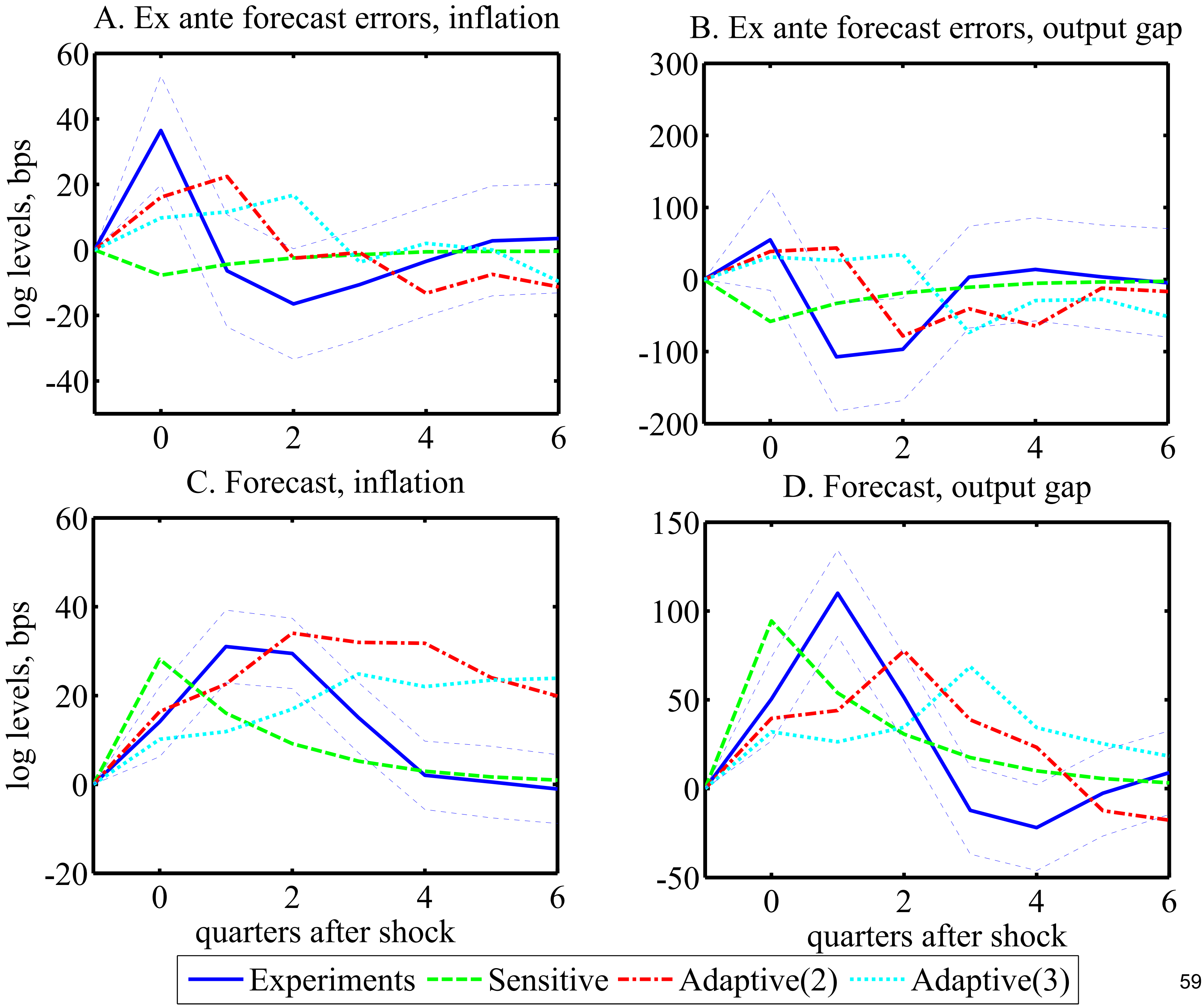
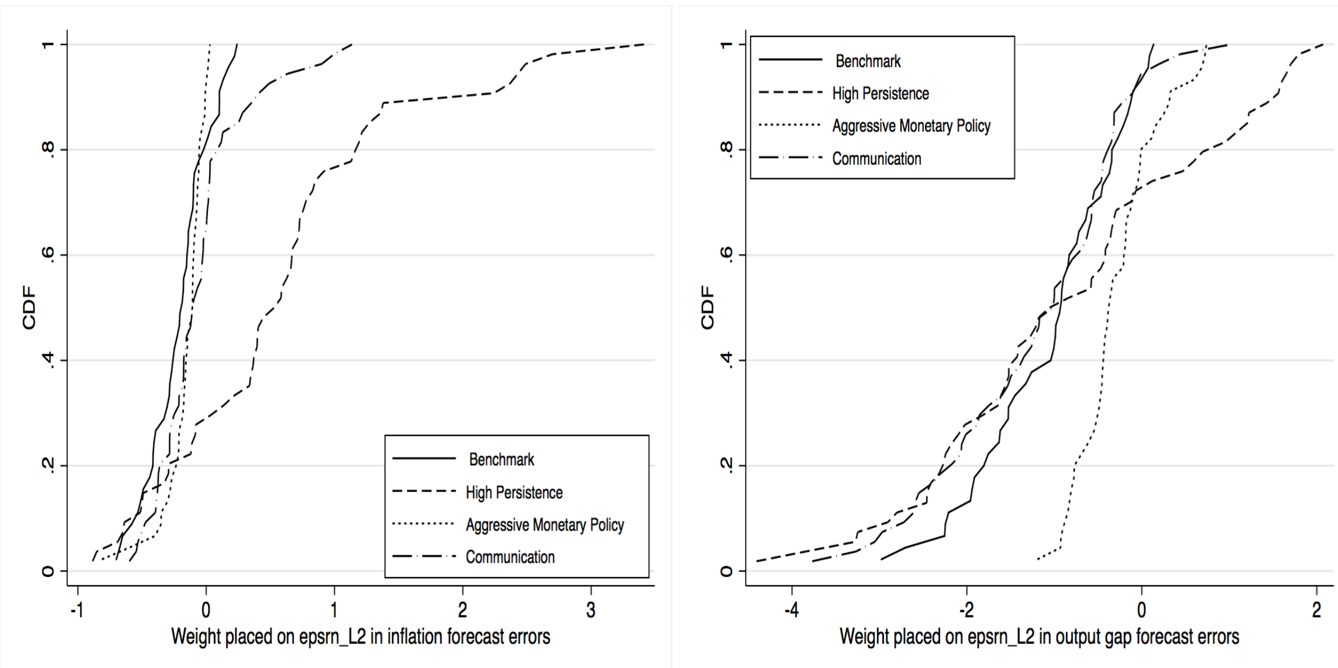
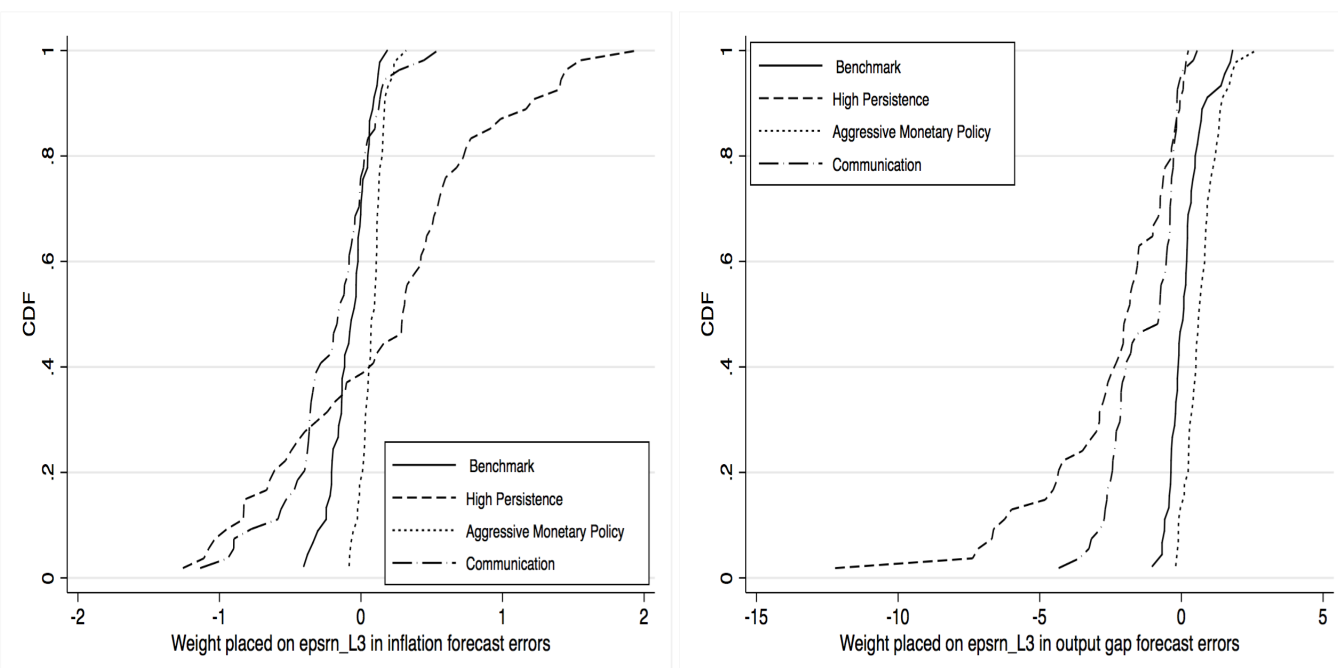


Figure A.5.: Distribution of Subject Forecast Error Responses to Lagged Innovations, Repetition 2, By Treatment

Panel A: Response of forecast errors to $_{t-2}$

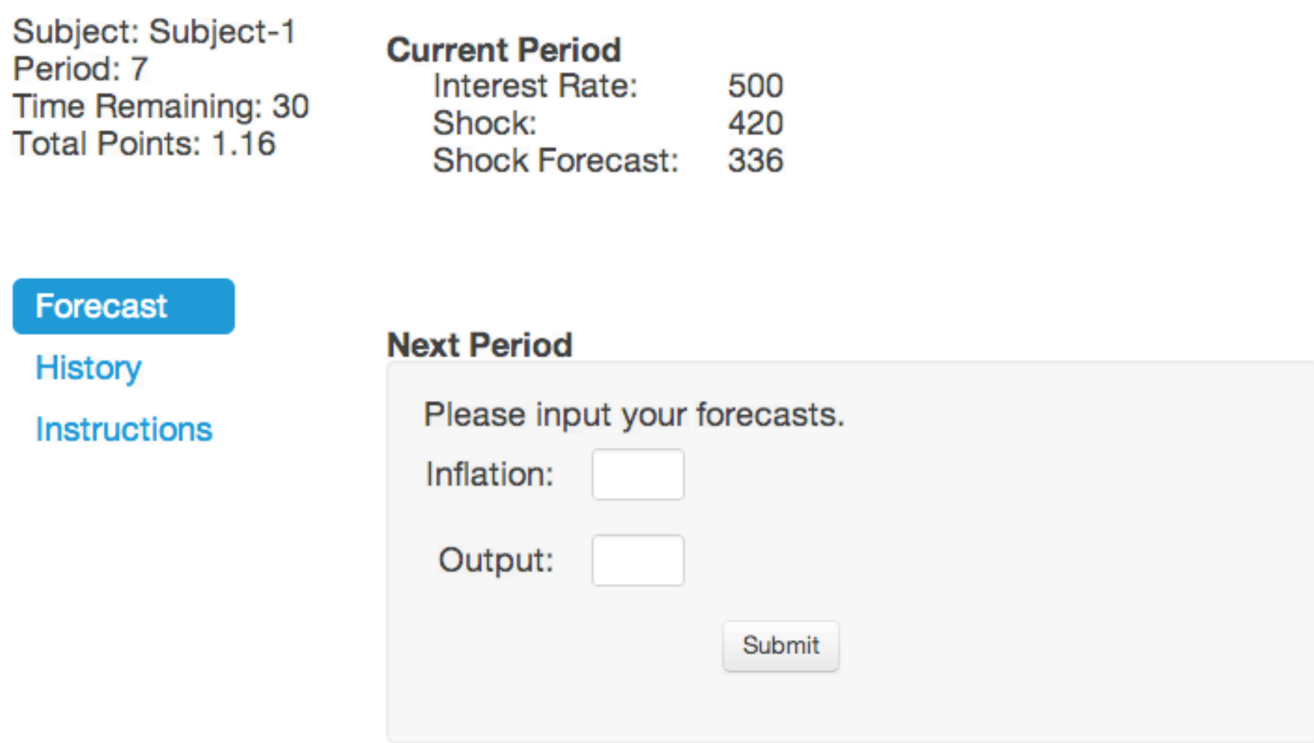


Panel B: Response of forecast errors to ε_{t-3}



Box A.1: Experimental interface, forecast screen

Forecast Screen



Box A.2: Experimental interface, history screen

History Screen

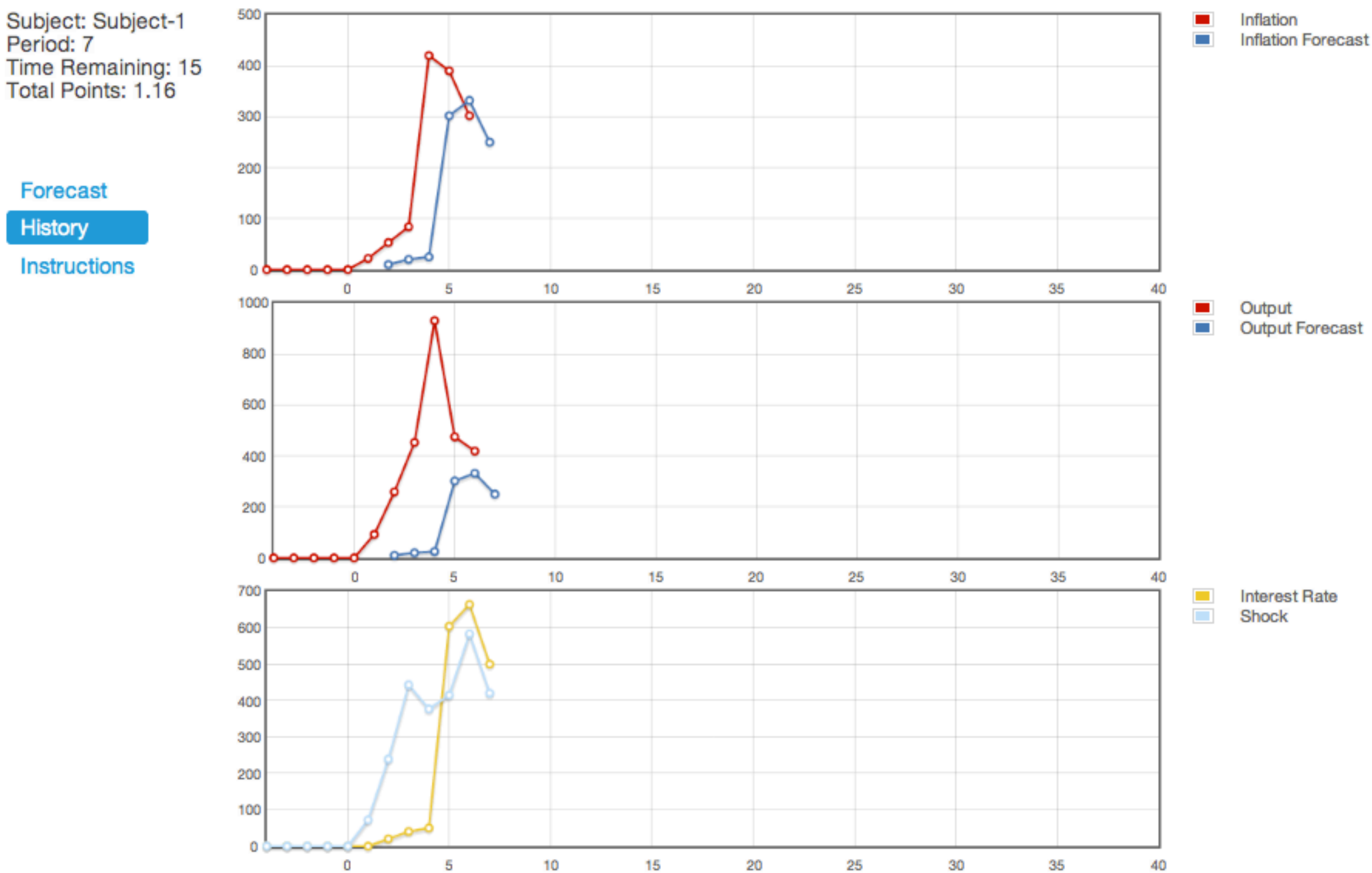


Table A.3: Non-Technical Instructions

Experimental Instructions

Welcome! You are participating in an economics experiment at CIRANO Lab. In this experiment you will participate in the experimental simulation of the economy. If you read these instructions carefully and make appropriate decisions, you may earn a considerable amount of money that will be immediately paid out to you in cash at the end of the experiment.

Each participant is paid CDN$10 for attending. Throughout this experiment you will also earn points based on the decisions you make. Every point you earn is worth $0.75.

During the experiment you are not allowed to communicate with other participants. If you have any questions, the experimenter will be glad to answer them privately. If you do not comply with these instructions, you will be excluded from the experiment and deprived of all payments aside from the minimum payment of CDN $10 for attending.

The experiment is based on a simple simulation that approximates fluctuations in the real economy. Your task is to serve as private forecasters and provide real-time forecasts about future output and inflation in this simulated economy. The instructions will explain what output, inflation, and the interest rate are and how they move around in this economy, as well as how they depend on forecasts. You will also have a chance to try it out in a practice demonstration.

In this simulation, households and firms (whose decisions are automated by the computer) will form forecasts identically to yours. So to some degree, outcomes that you will see in the game will depend on the way in which all of you form your forecasts. Your earnings in this experiment will depend on the accuracy of your individual forecasts.

Below we will discuss what inflation and output are, and how to predict them. All values will be given in basis points, a measurement often used in descriptions of the economy. All values can be positive, negative, or zero at any point in time.

INFORMATION SHARED WITH ALL PARTICIPANTS

Each period, you will receive the following information to help you make forecasts.

Interest Rate

The interest rate is the rate at which consumers and firms borrow and save in this experimental economy. The central bank that sets the interest rate is forward-looking in that it responds to forecasts of future inflation and output. It will aim to keep inflation and output equal to zero.

Depends on: **Forecasted inflation for current period (+)**

Example: If the median subject forecasts inflation to be positive in the next period, the interest rate next period will be positive. If the median forecast for inflation is negative, the interest rate will be negative.

Depends on: **Forecasted output for current period (+)**

Example: If the median subject forecasts output to be positive in the next period, the interest rate next period will be positive. If the median forecast for inflation is negative, the interest rate will be negative.
Question: If the median forecasts for inflation and output are -10 and -20, respectively, what sign will the interest rate be? _________. What if the median forecast for inflation is -10 and output is 20?________

Current Shock

A shock is a random "event" that affects output. E.g. A natural disaster can suddenly destroy crops, or a technological discovery immediately improves productivity.

Depends on: **Random Draw**

The shock will be relatively small most of the time. Two-thirds of the time it will fall between -138 and 138 points, and 95% of the time it will fall between -276 and 276 points. On average, it will be 0. (But rarely will it ever be exactly zero!)

Every shock takes some time to dissipate. Suppose the shock in the current period is 100. Next period, that shock will now be 57% of 100, or 57 points. Assuming no new shocks were to occur, the value of the shock next period is 57 points. Some shock is likely to occur.

Shock Forecast

The shock forecast is a prediction of what the shock will be next period. It assumes that, on average, next period's shock is zero.

Example: If the current shock is -200 points, the forecasted value of the shock tomorrow is -200(0.57) = -114

HOW INFLATION AND OUTPUT ARE DETERMINED

You will be making forecasts about what you believe inflation and output will be tomorrow.

1. ***Inflation***

Inflation is the rate at which overall prices change between two periods.

Depends on: **Forecasted inflation in the next period (+)**

Example: If the median subject forecasts future inflation to be positive, current inflation will be positive, and vice versa.

Question: Holding all else constant, will current inflation be positive or negative if the median forecast for future inflation is -20? _________________.

Current output (+)

Example: If current output is positive, current inflation will be positive. If current output is negative, current output will be negative.

Question: Holding all else constant, what sign is current inflation if current output is 50?______0?_________

2. ***Output***

Output refers to a measure of the quantity of goods produced in a given period.

Depends on: **Forecasted output in the next period (+)**

Example: If the median subject forecasts future output to be positive, current output will also be positive.

Question: Holding all else constant, will output be positive or negative if the median subject forecasts output to be -15 points next period?_____________

Forecasted inflation in the next period (+)

Example: If the median subject forecasts inflation to be positive next period, current output will be positive.

Question: Holding all else constant, what sign will output be if the median subject forecasts inflation to be 250 points next period?_____________. What sign will inflation have? ___________.

Current interest rate (-)

Example: If the current interest rate is positive, current output will be positive.

Question: Holding all else constant, what sign will output be if interest rates are 10? ______________ What sign will inflation have? _____________

Random Shocks (+)

Example: Positive shocks will have a positive effect on output. Negative shocks will have a negative effect on output.

Question: Holding all else constant, what sign will output be if the shock is -50?_________ . What sign will inflation have? ________________

Score

Your score will depend on the accuracy of your forecasts. The absolute difference between your forecasts and the actual values for output and inflation are your absolute forecast errors.

Absolute Forecast Error = absolute (Your Forecast – Actual Value)
Total Score = 0.30(2^-0.01 (Forecast Error for Output)) + 0.30(2^-0.01 (Forecast Error for Inflation))

The maximum score you can earn each period is 0.60.
Your score will decrease as your forecast error increases. Suppose your forecast errors for each of output and inflation are:

0	-Your score will be 0.6	300	-Your score will be 0.075
50	-Your score will be 0.42	500	-Your score will be 0.02
100	-Your score will be 0.30	1000	-Your score will be 0
200	-Your score will be 0.15	2000	-Your score will be 0

Information about the Interface, Actions, and Payoffs

During the experiment, your main screen will display information that will help you make forecasts and earn more points.

At the top left of the screen, you will see your subject number, the current period, time remaining-, and the total number of points earned.

Below that you can click on different tabs to access different information. These tabs are the Forecast Tab, History Tab, and Instructions Tab.

On the Forecast Tab, you will see information that is common to all participants: the current interest rate, the size of the shock to output, and the forecasted shock for next period.

When the period begins, you will have 60 seconds to submit new forecasts for the next period's inflation and output levels. You may submit both negative and positive forecasts. Please review your forecasts before pressing the SUBMIT button. Once the SUBMIT button has been clicked, you will not be able to revise your forecasts until the next period. You will earn zero points if you do not submit both forecasts. The amount of time will be reduced to 45 seconds in later periods.

On the History tab, you will see three history plots. The top history plot displays your past forecasts of output and the realized output levels. The second plot displays your past forecast of inflation and realized inflation levels. The difference between your forecasts and the actual realized levels constitutes your forecast errors. Your forecasts will always be shown in blue while the realized value will be shown in red. The final plot displays past interest rates and the shock to output. You can see the exact value for each point on a graph by placing your mouse at that point.

On the Instructions Tab, you may view a more technical version of these instructions.

Each economy will last for between 50-60 periods. The environment will then be reset such that inflation, output, and interest rates return to zero. A new economy will begin and your previous decisions will not play a role. Your scores from each of the economies plus the show up fee will be paid to you in cash at the end of the experiment.

Box A.4: Technical Instructions

The economy consists of four main variables:

- Inflation
- Output
- Interest rate
- Shocks

At any time, t, the values of these variables will be calculated as follows:

Interest $Rate_t$ = 1.5(Median forecast of $Inflation_t$ formed last period)
+0.5(Median forecast of $Output_t$ formed last period)

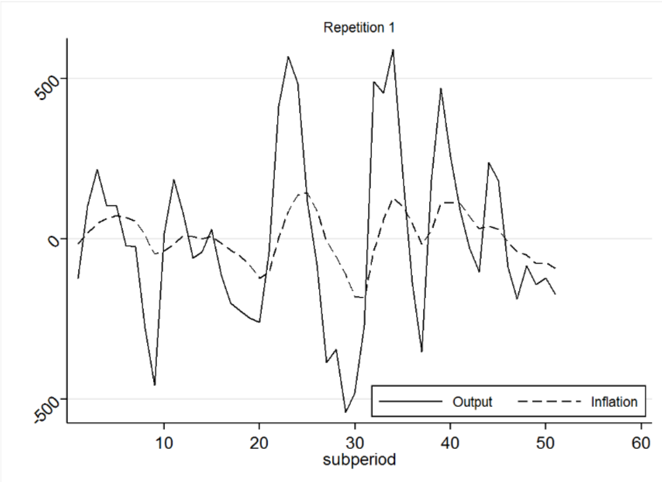
$Inflation_t$ = 0.989(Median forecast of $Inflation_{t+1}$)+0.13($Output_t$)

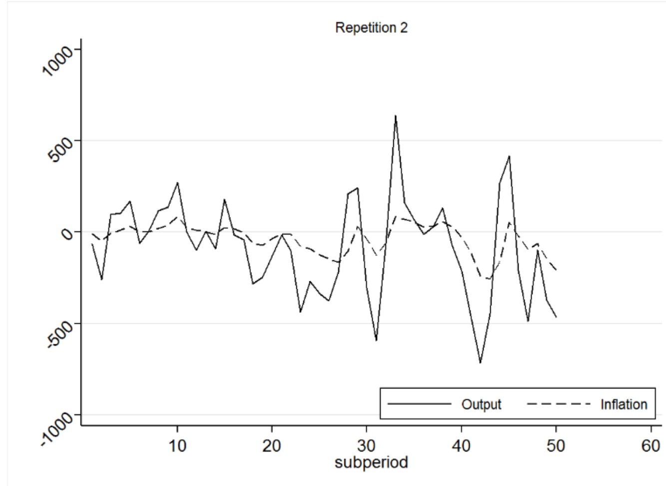
$Output_t$ = Median forecast of $Output_{t+1}$ + Median forecast of $Inflation_{t+1}$ - Interest $rate_t$ + $Shock_t$

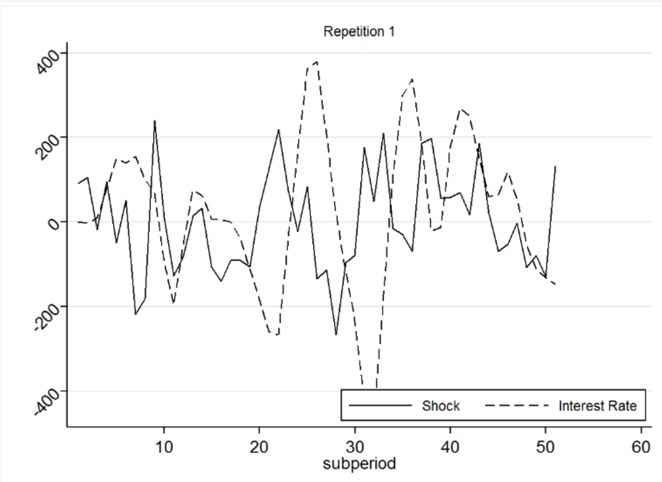
$Shock_t$ = 0.57($Shock_{t-1}$)+Random $component_t$

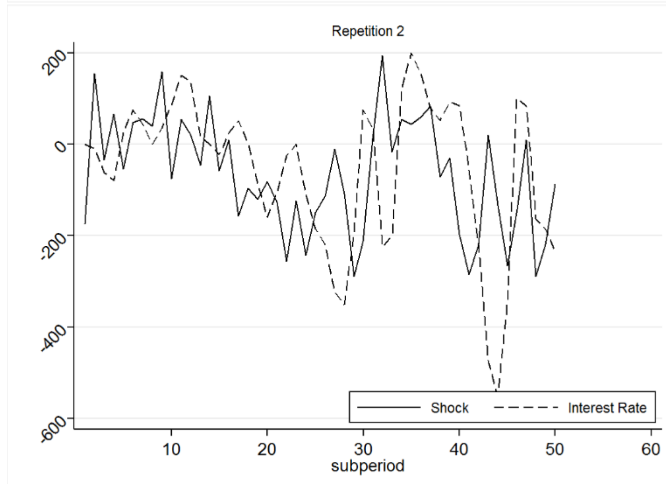
- The random component is 0 on average.
- Roughly two out of three times the shock will be between -138 and 138 basis points.
- 95 per cent of the time the shock will be between -276 and 276 basis points.

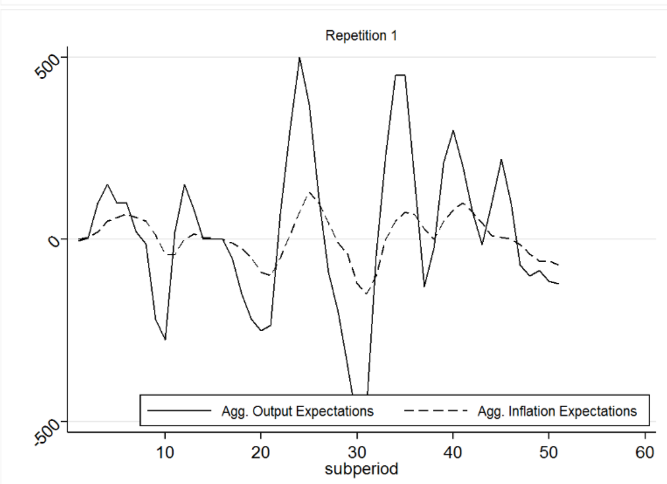
Benchmark – Session 1 Time Series Data

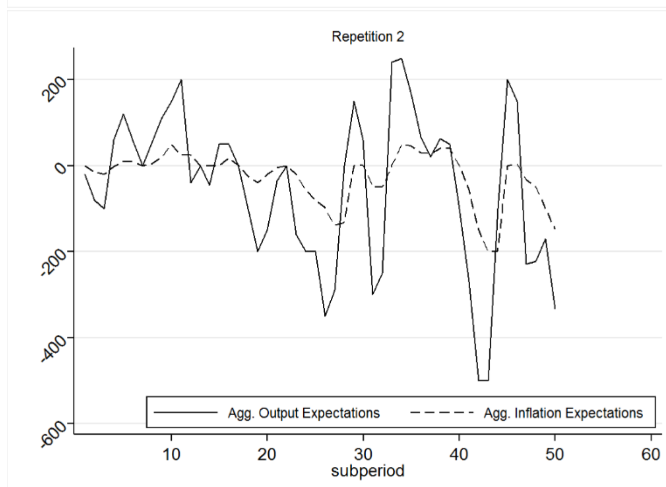




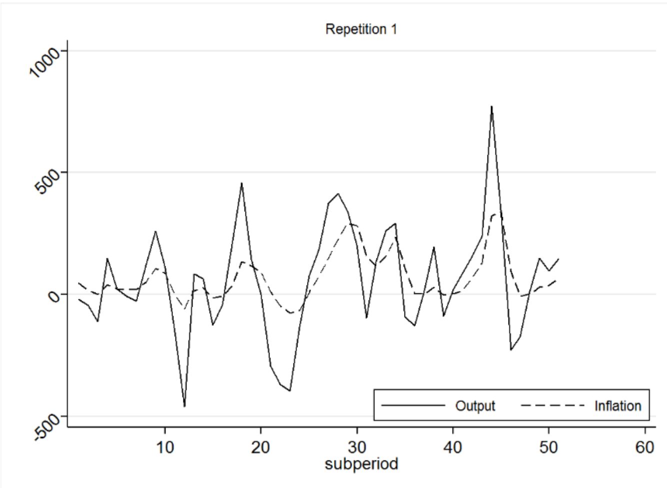


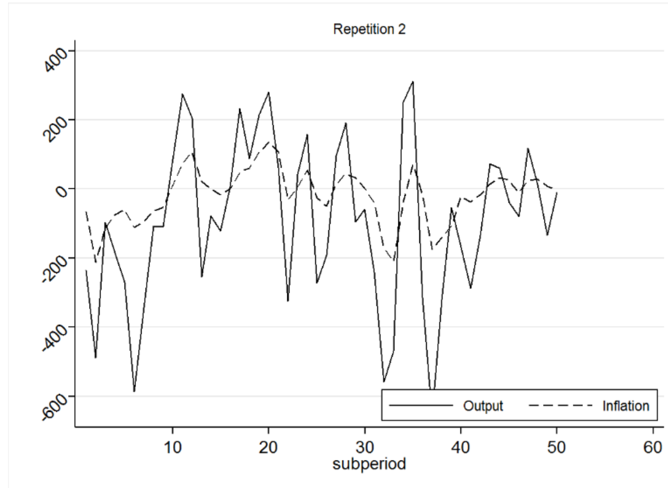


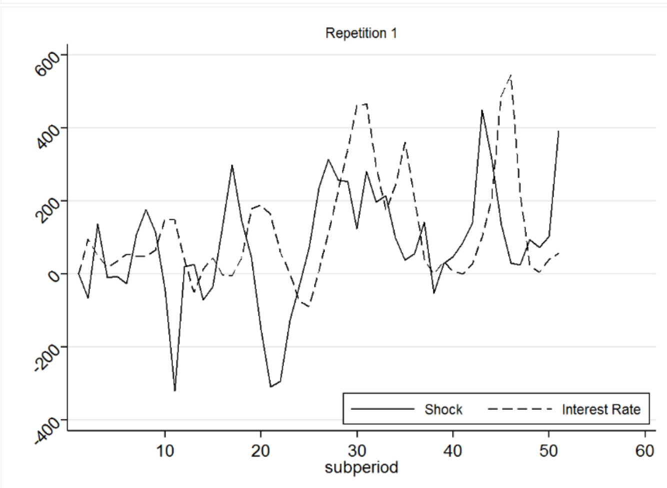


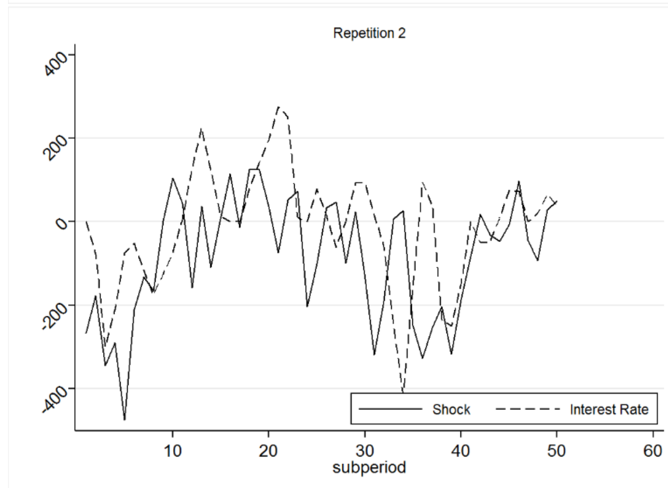


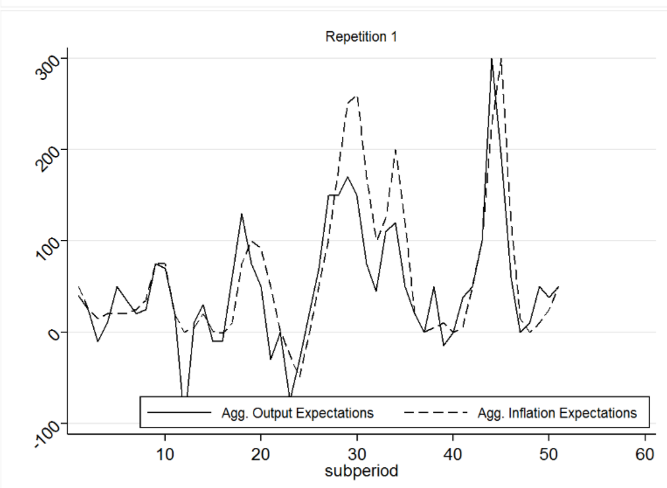
Benchmark – Session 2

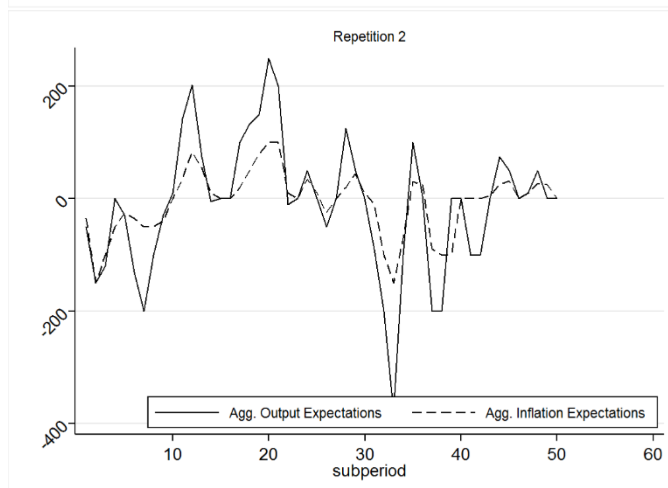




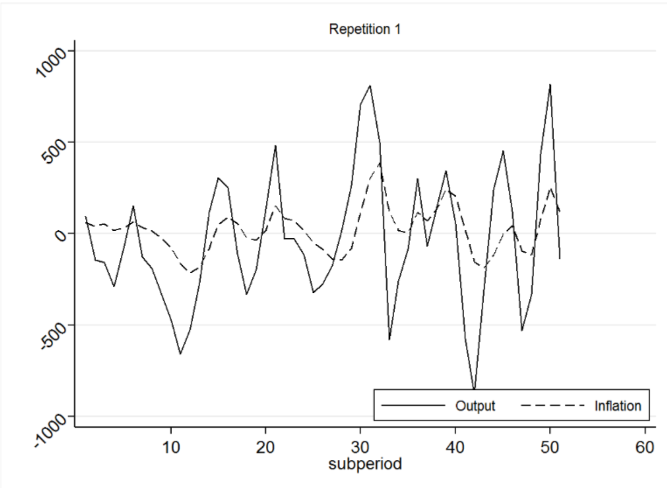


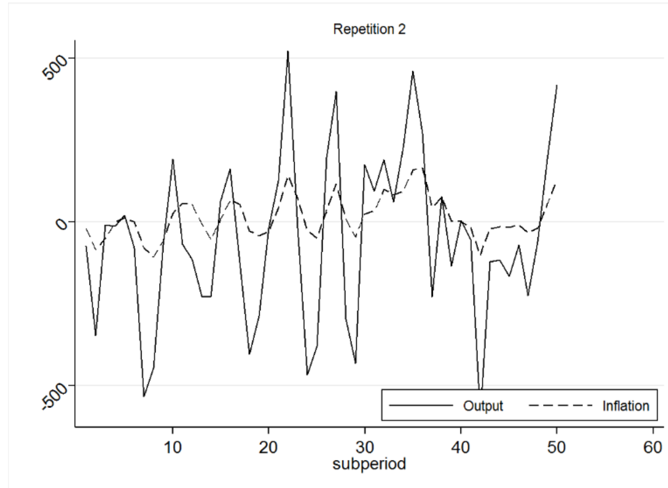


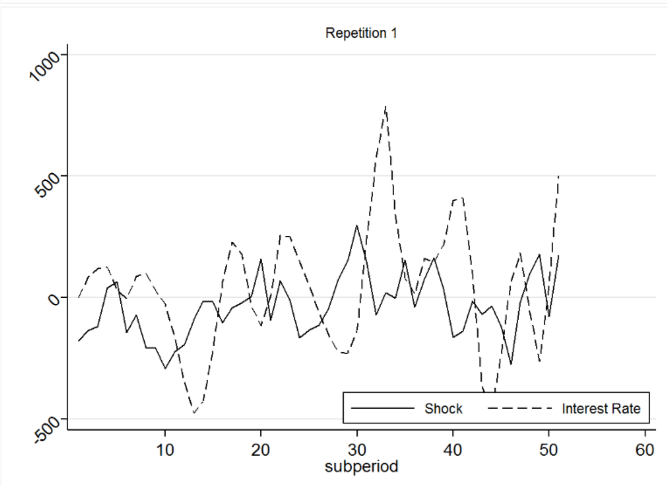


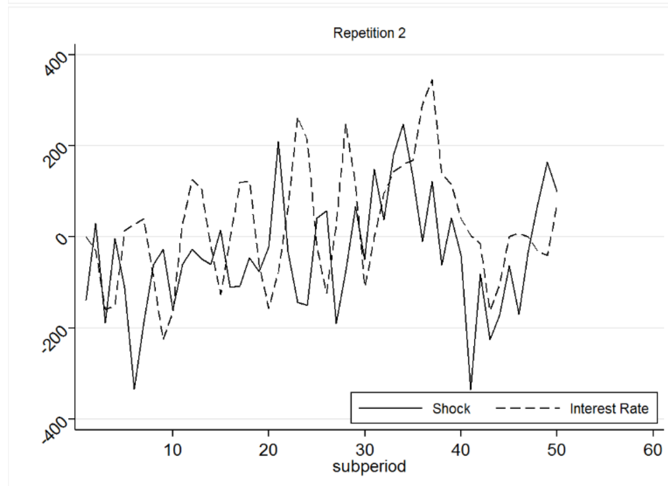


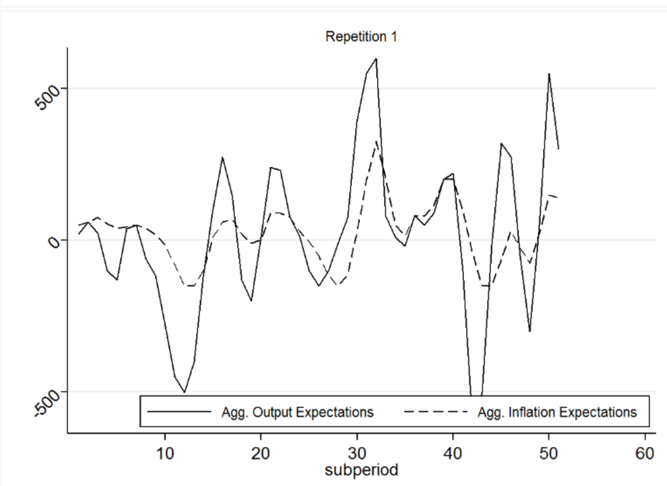
Benchmark – Session 3

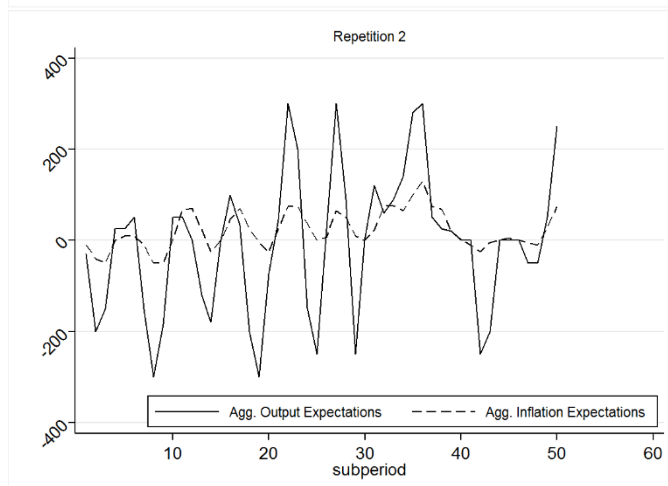












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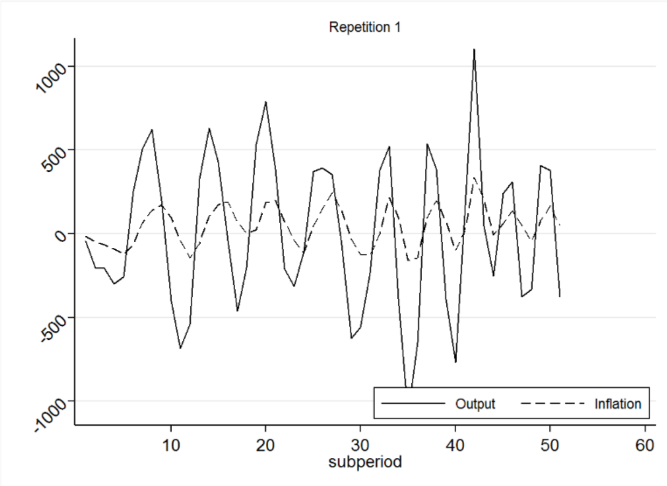
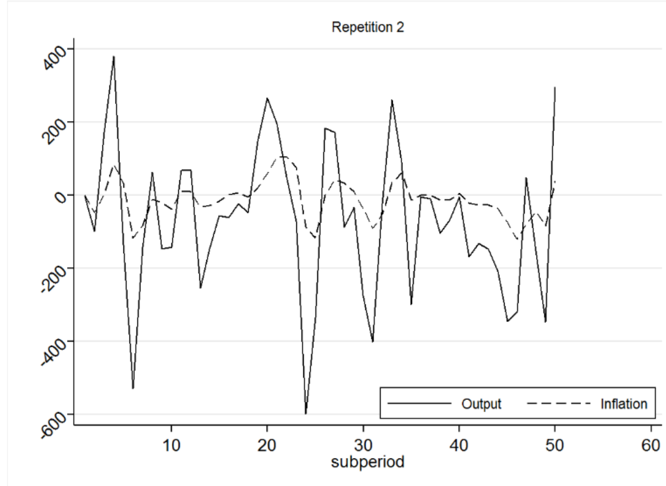
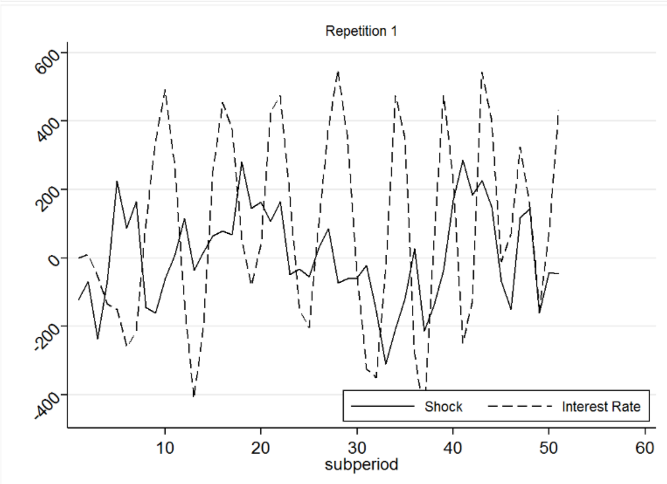
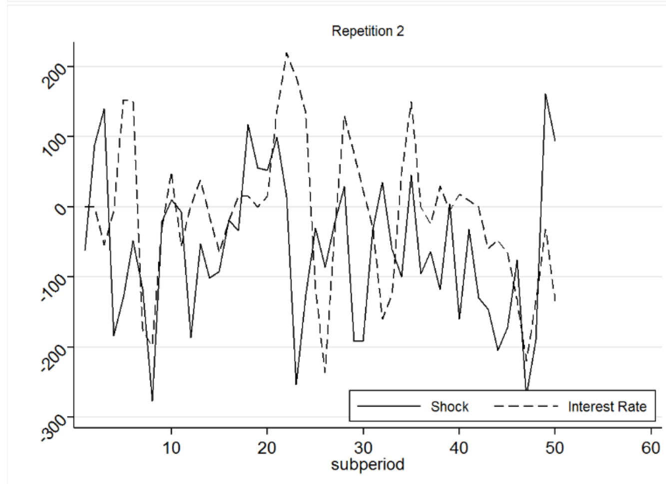
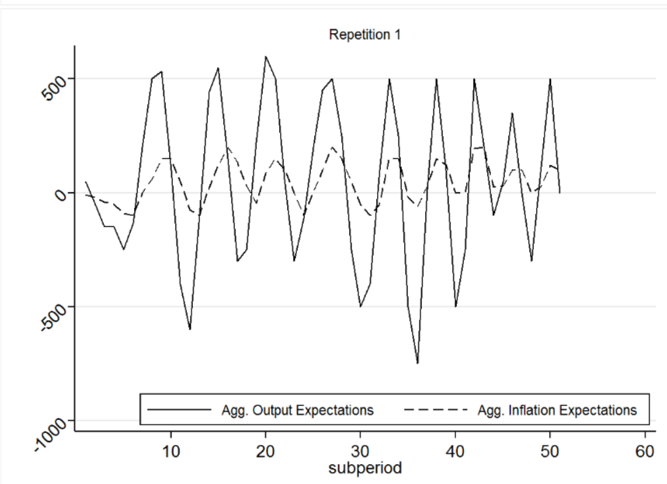
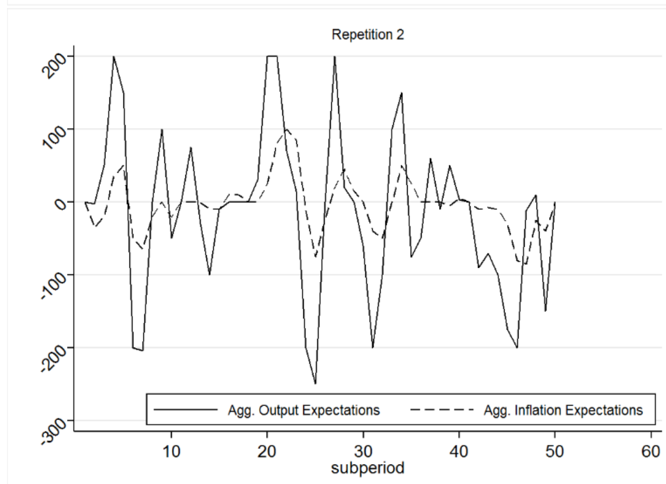




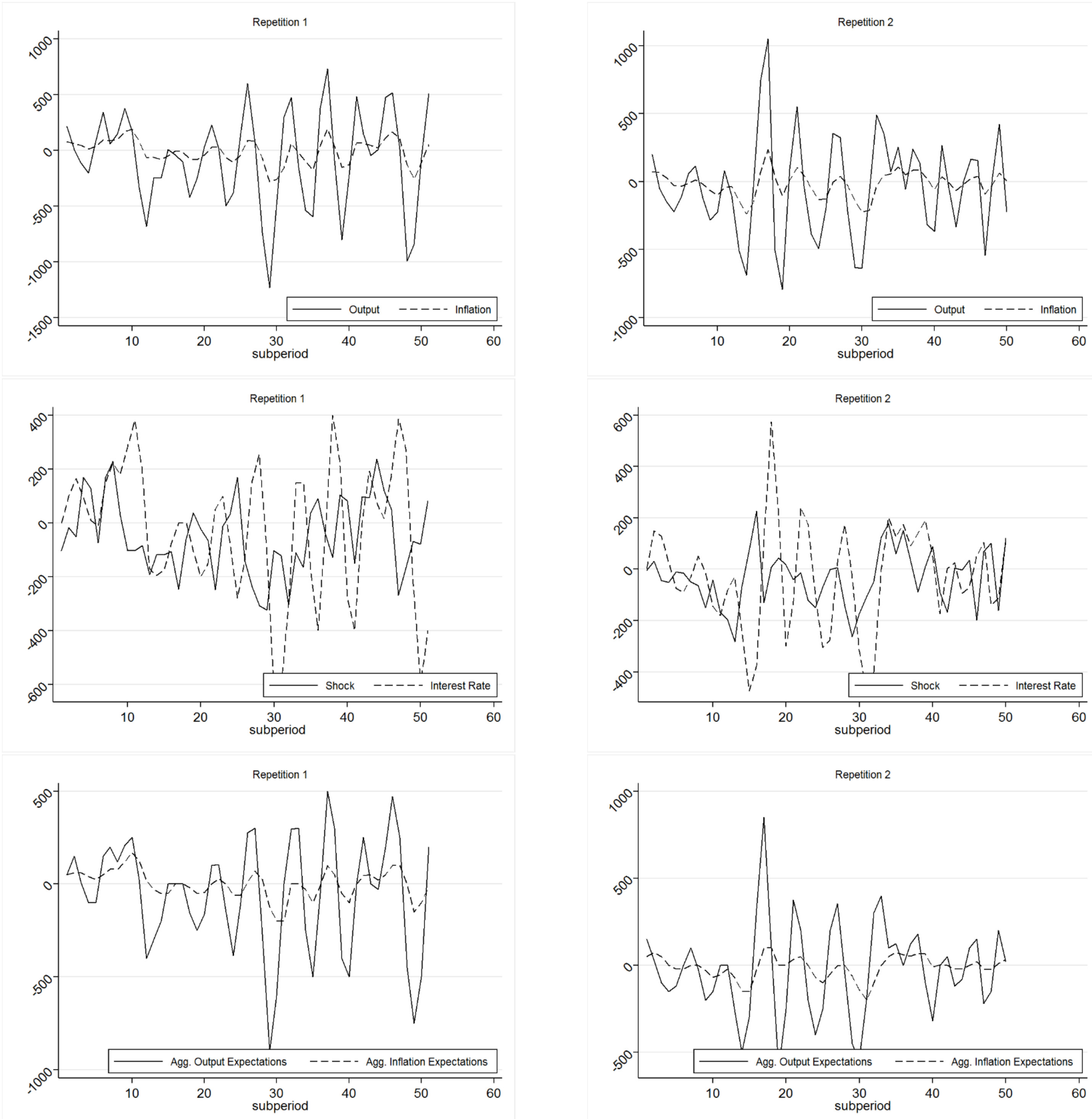




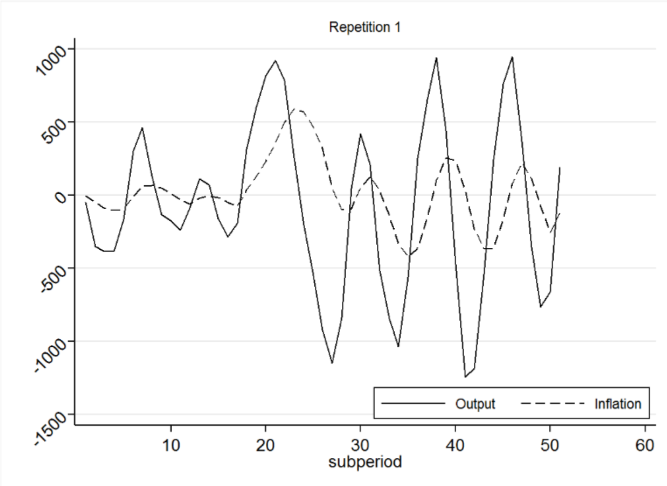


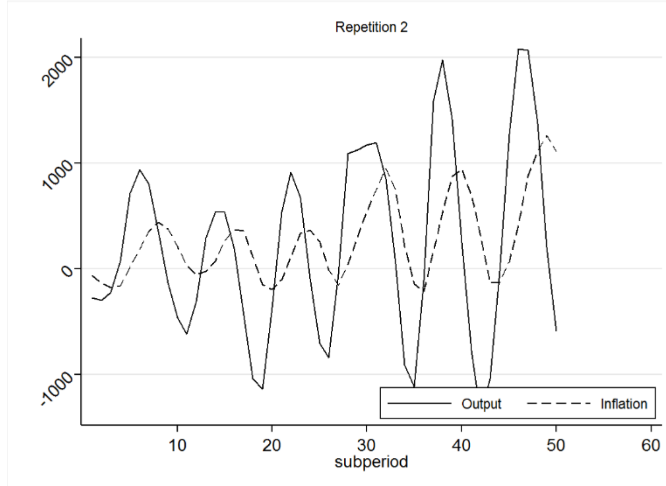


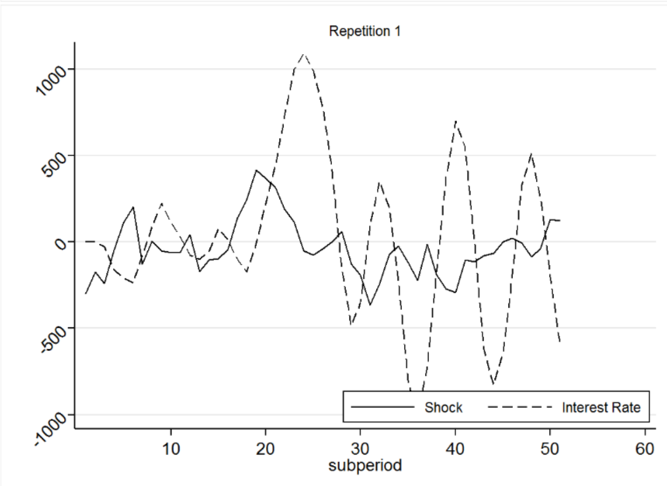
Benchmark – Session 5

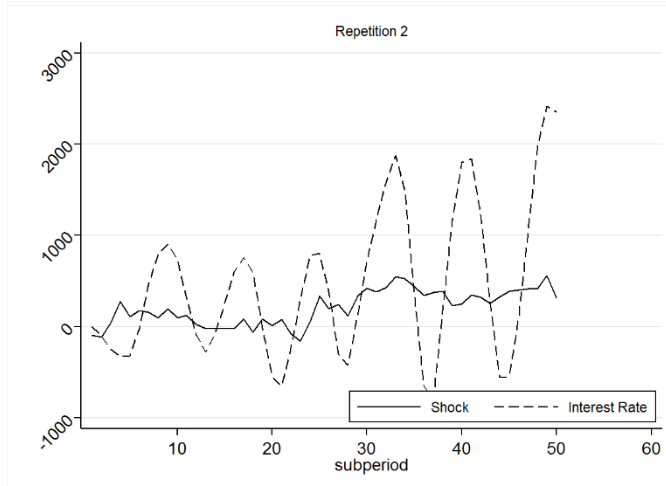


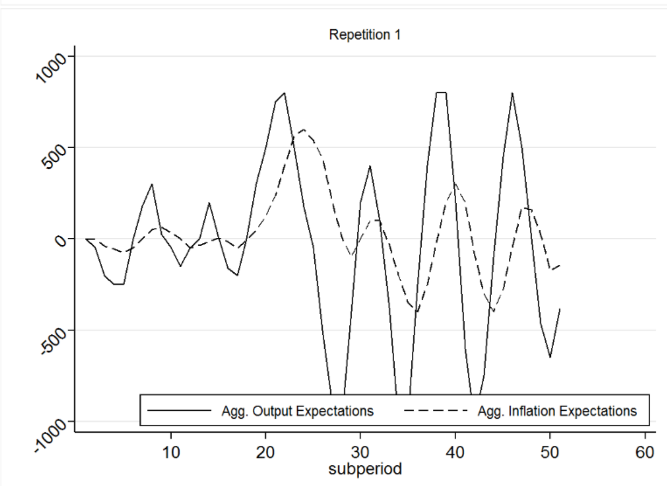
Shock – Session 1

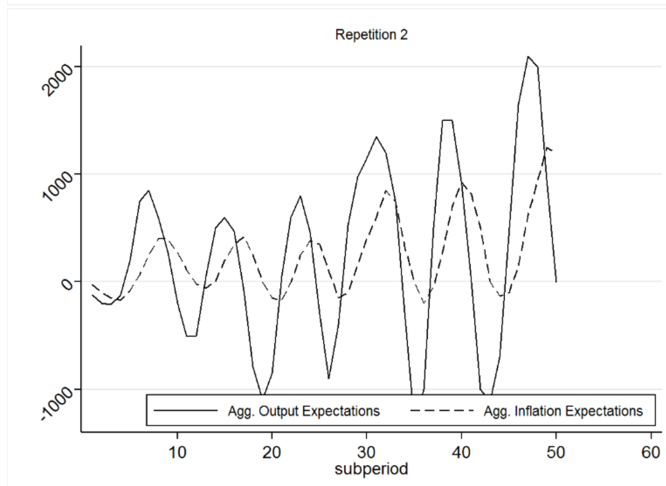




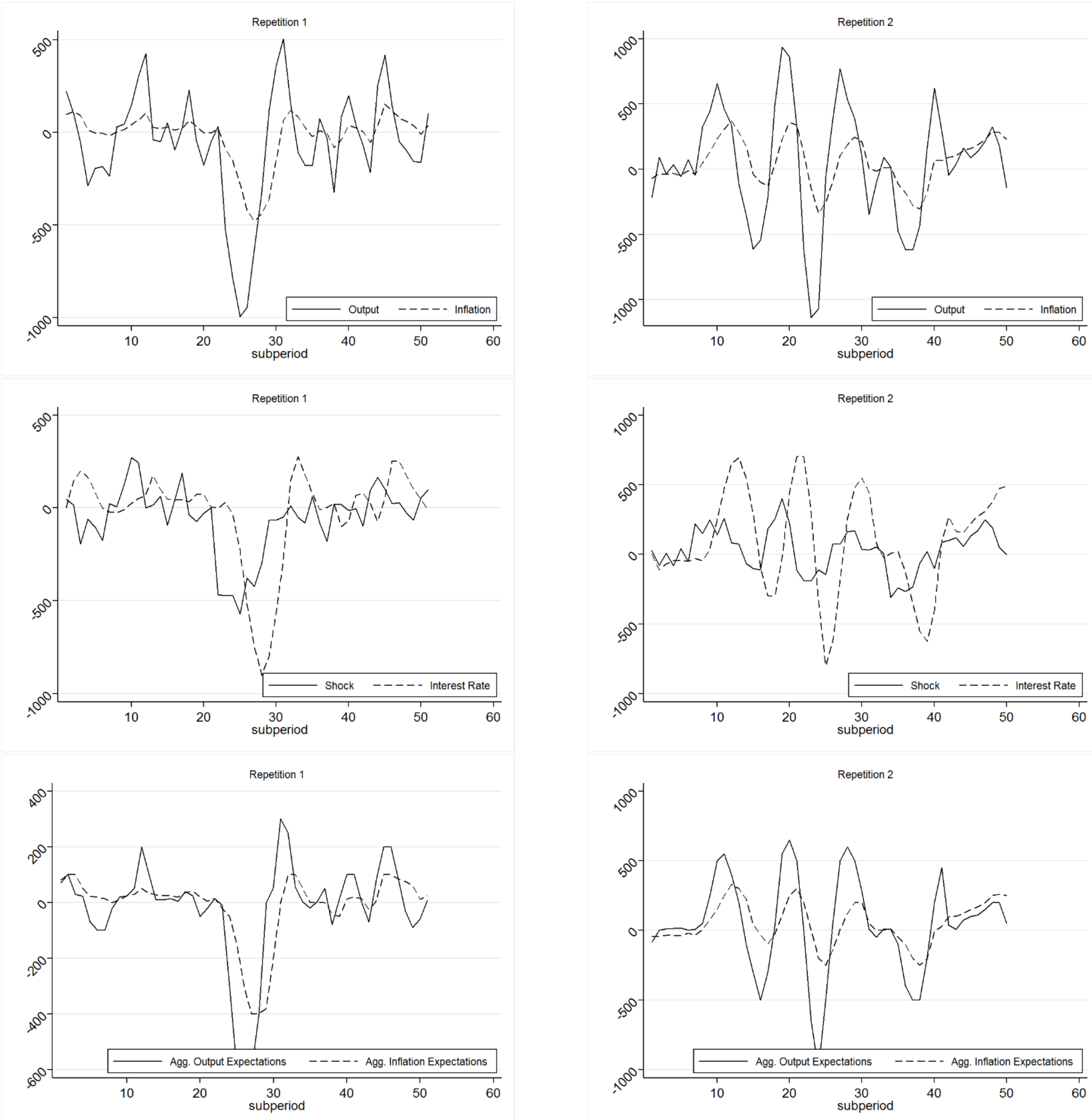




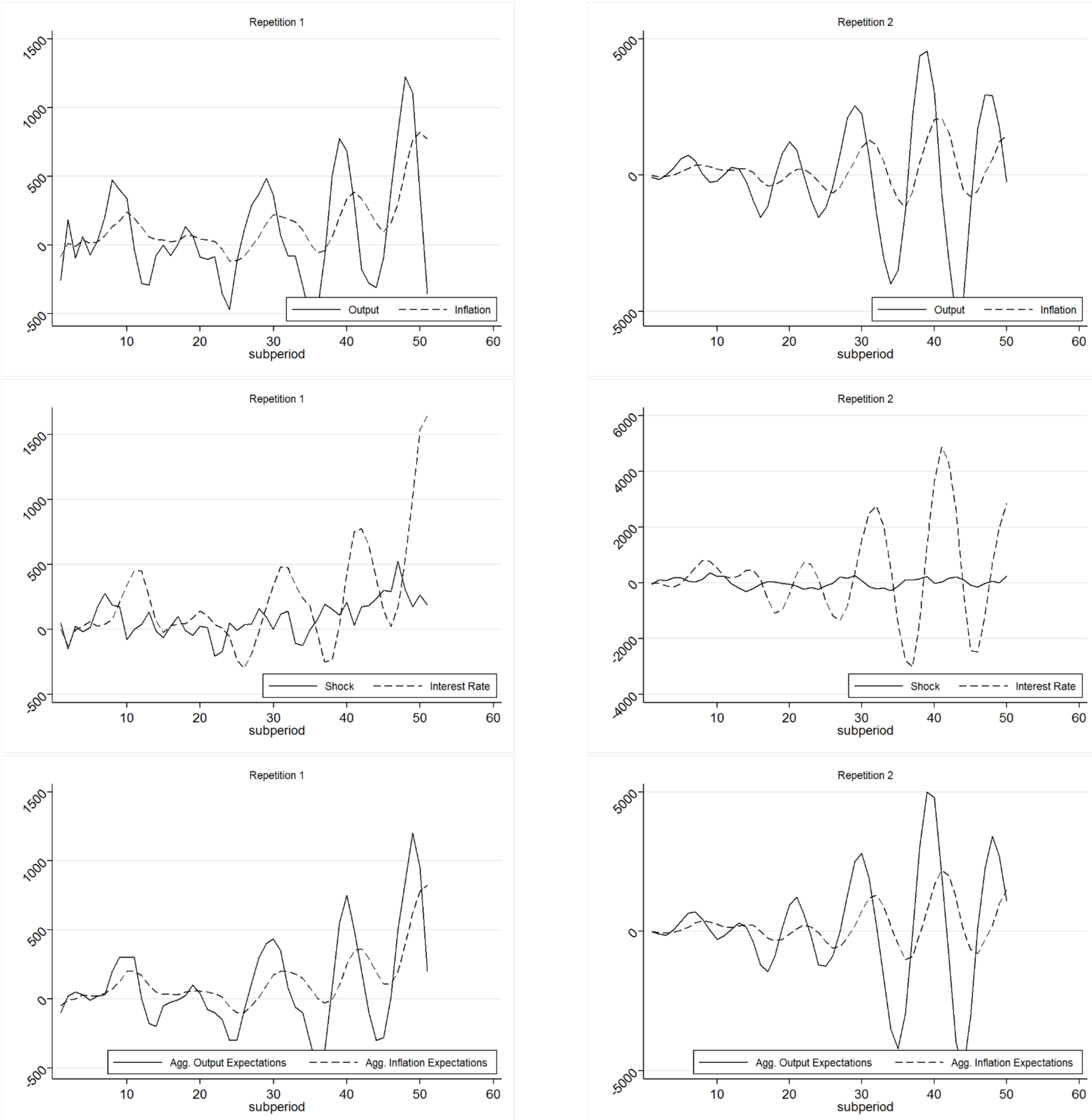




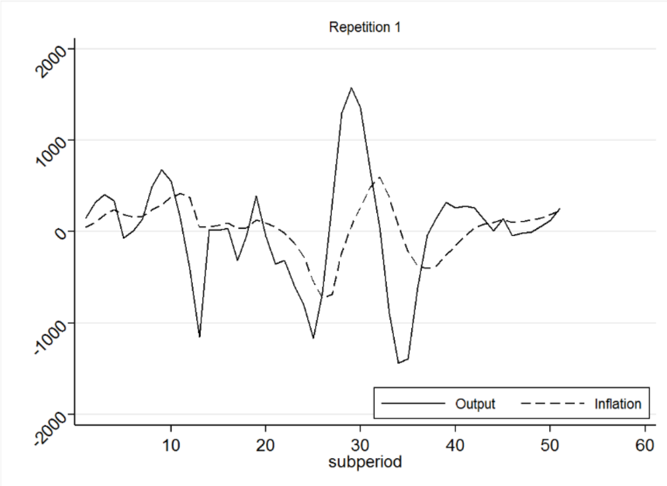
Shock – Session 2

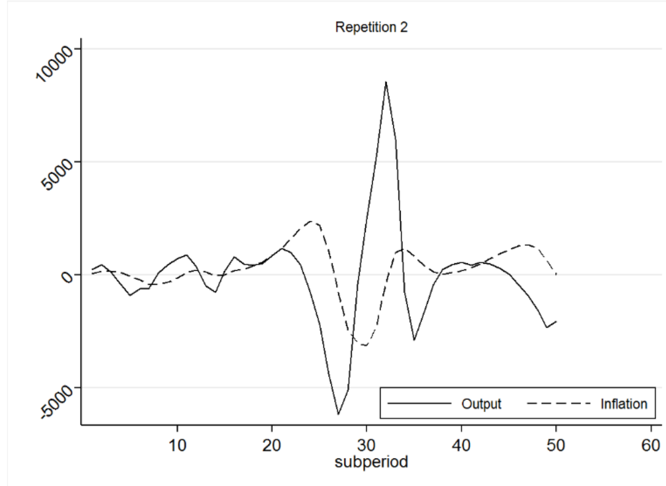


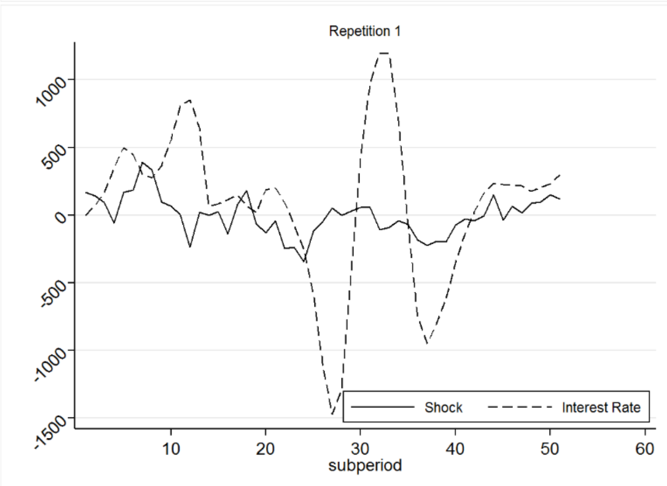
Shock – Session 3

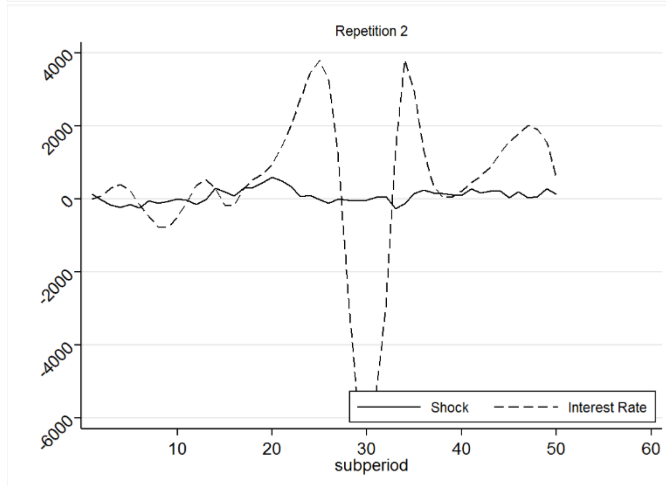


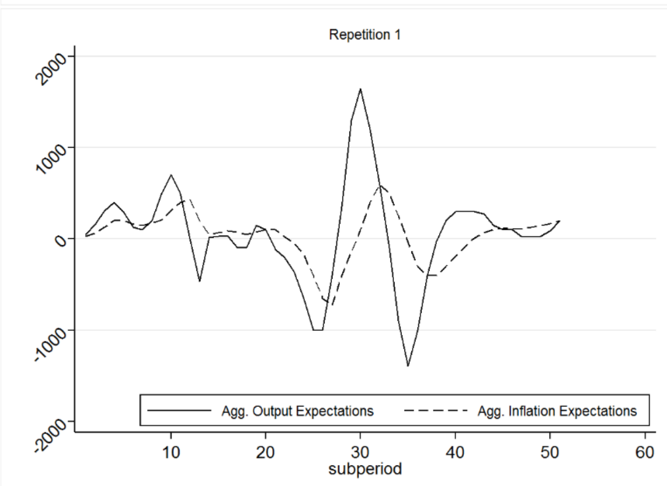
Shock – Session 4

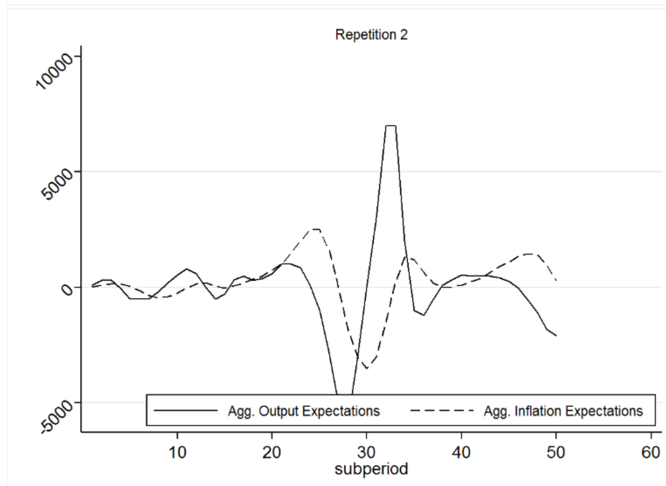




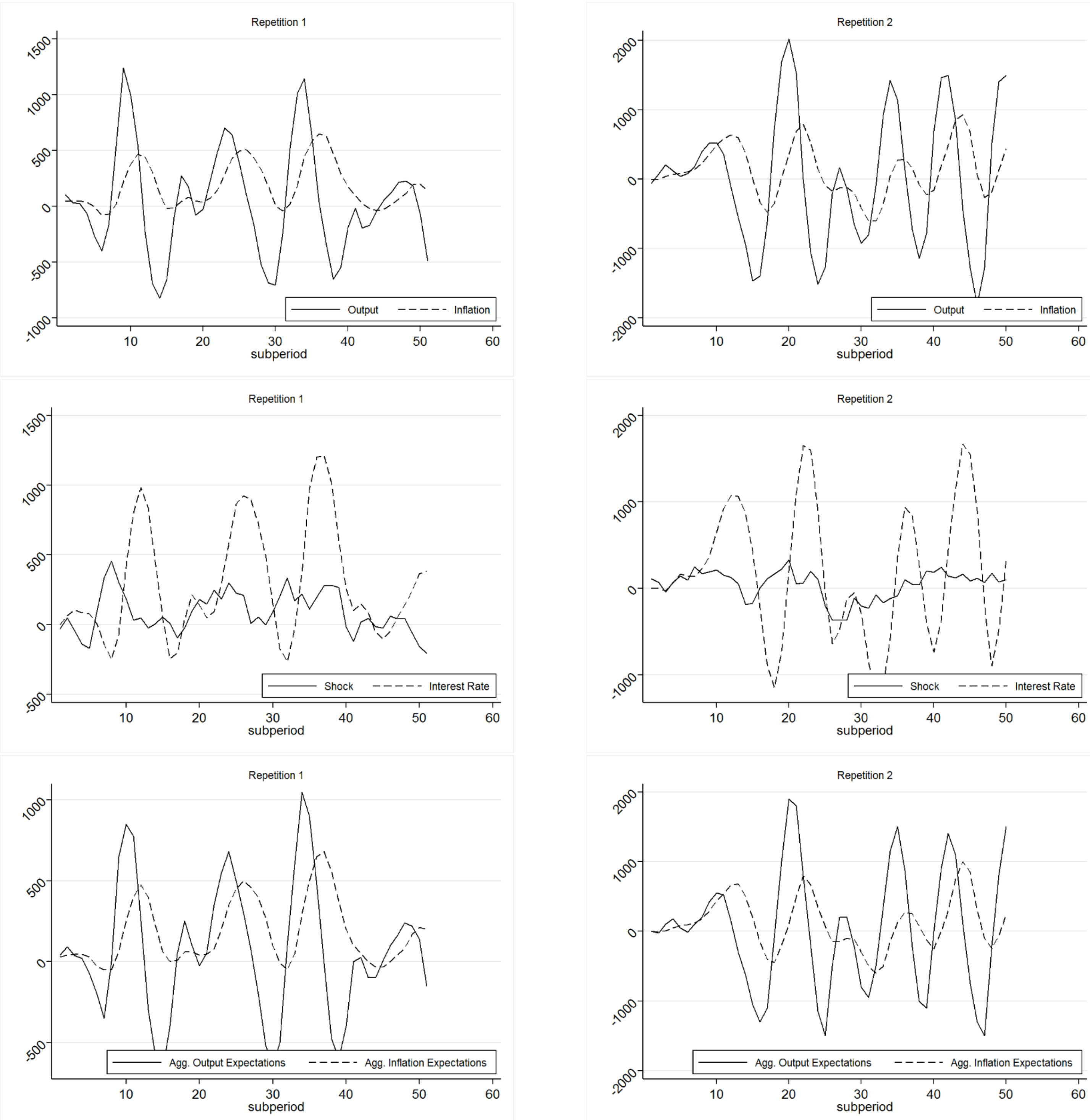




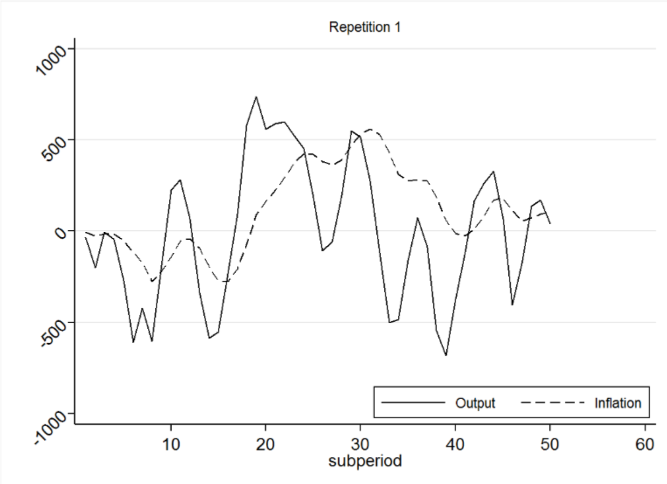


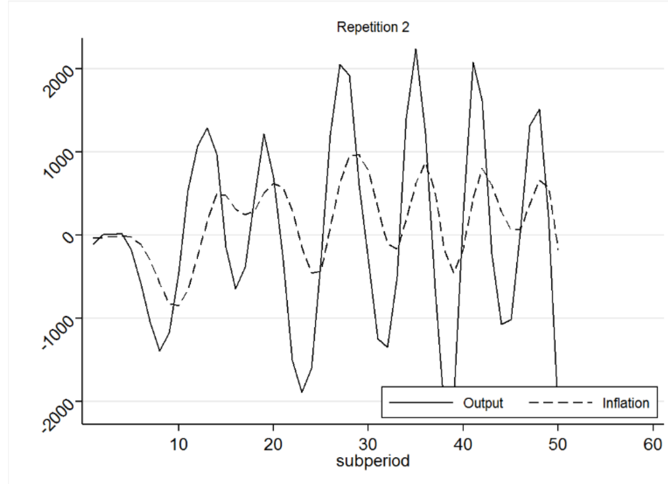


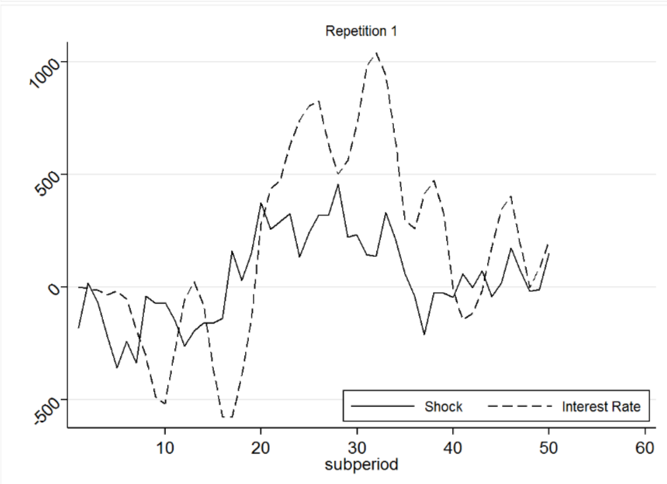
Shock – Session 5

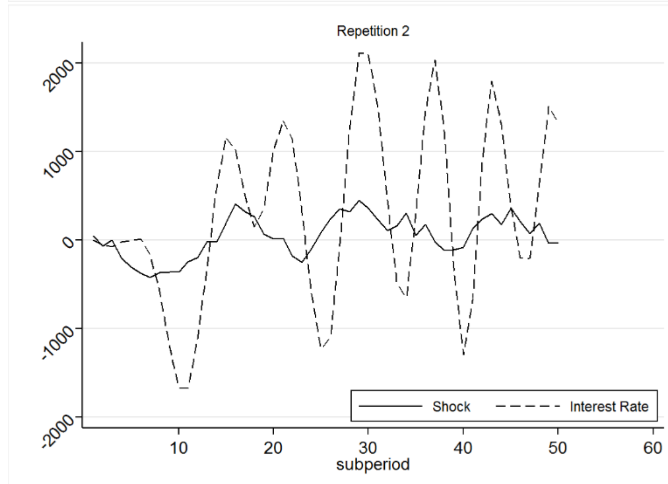


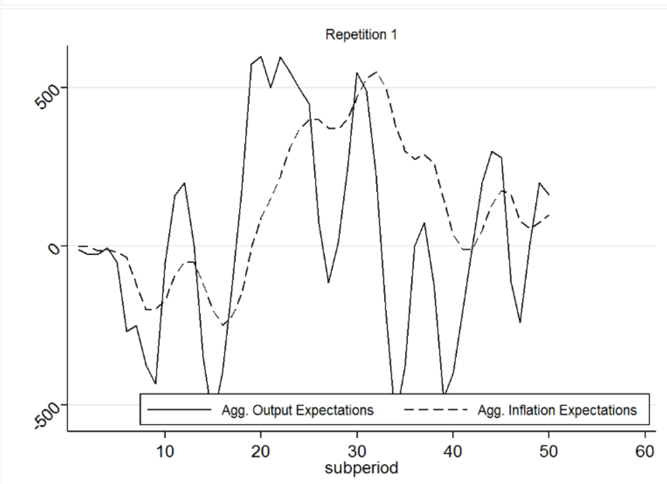
Shock – Session 6

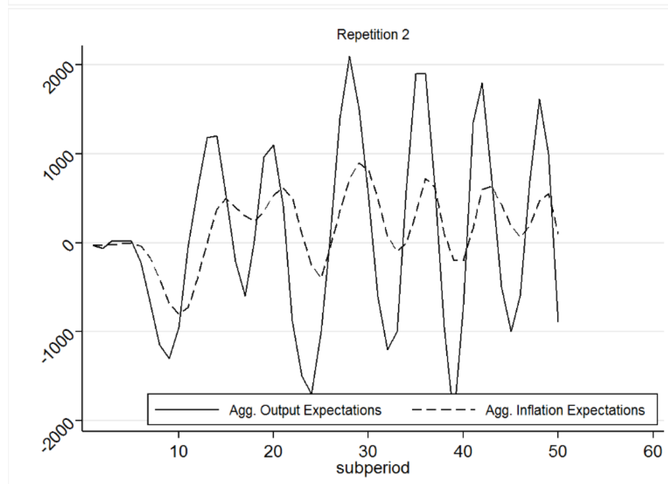




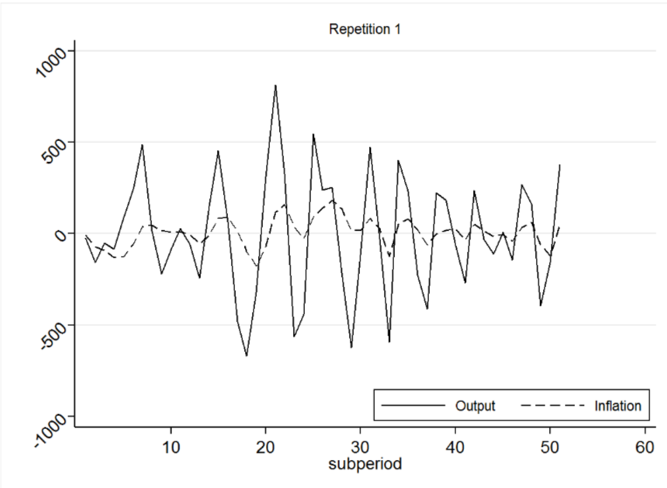


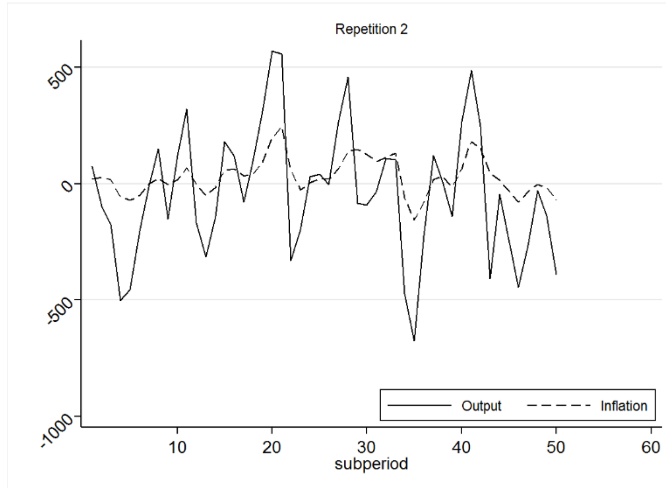


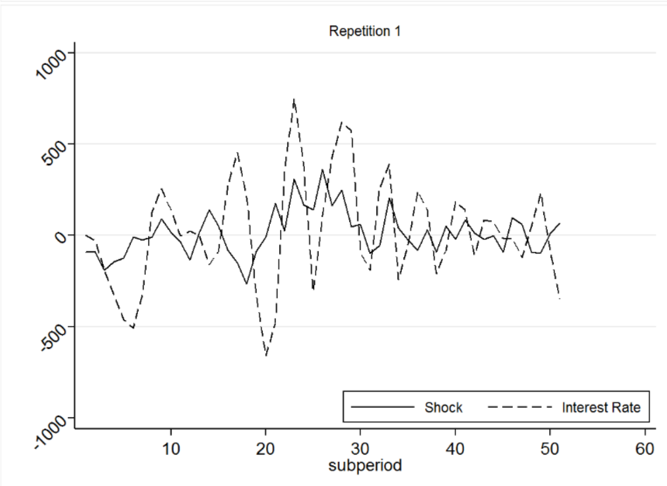


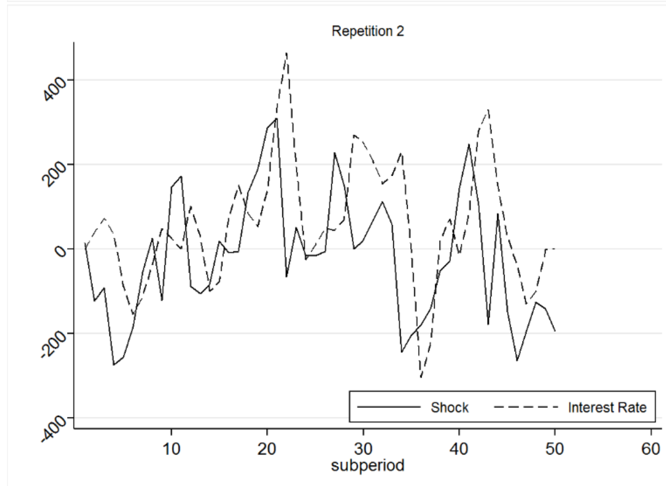


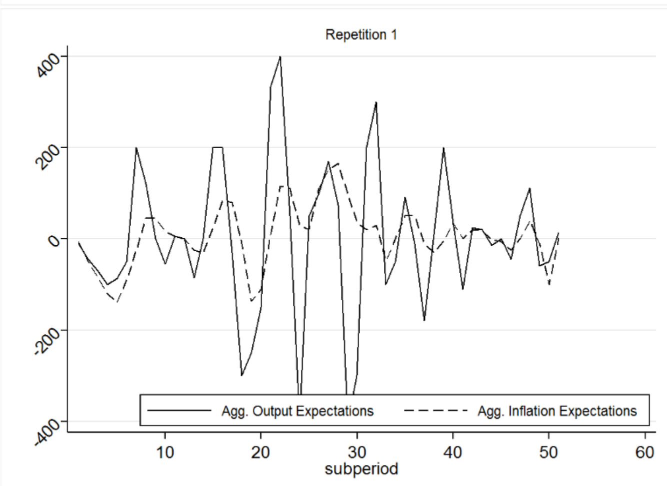
Aggressive Monetary Policy – Session 1

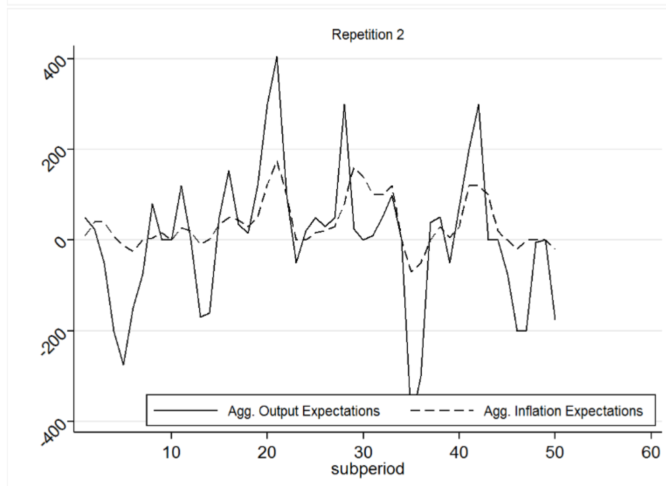




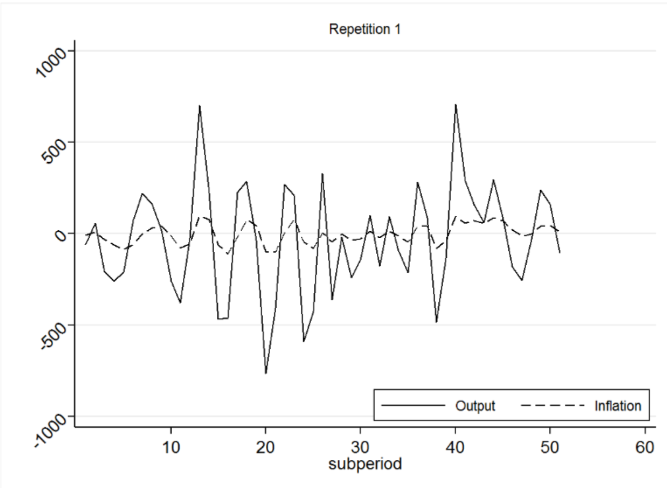


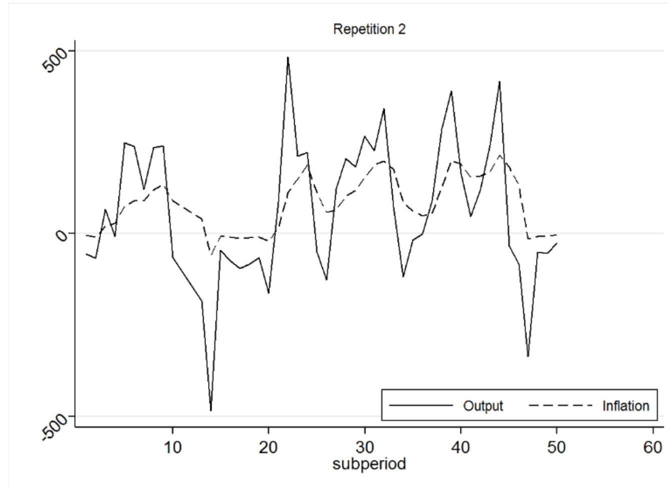


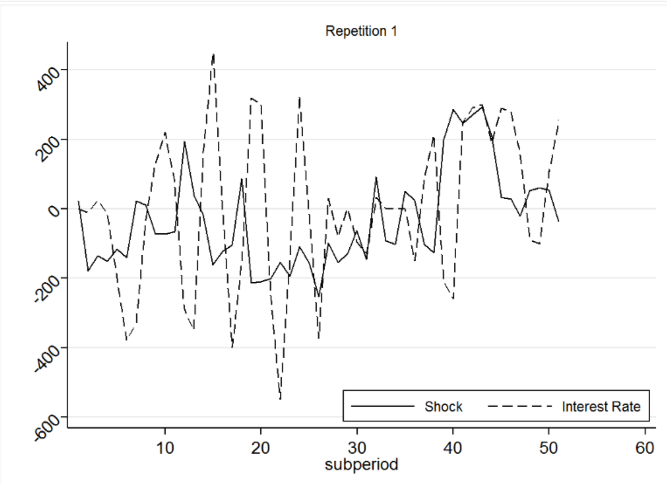


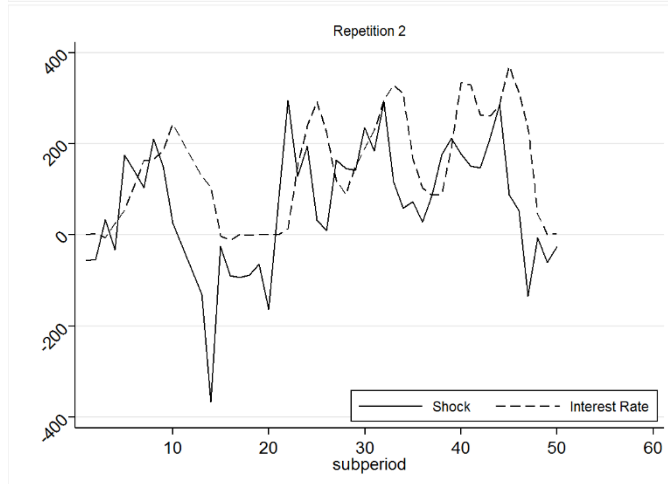


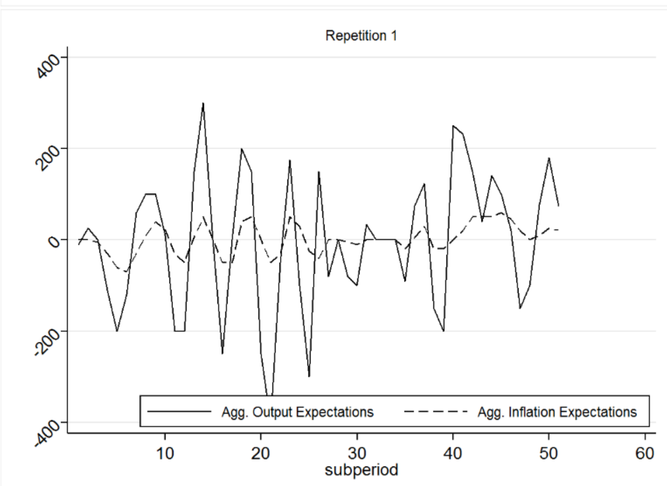
Aggressive Monetary Policy – Session 2

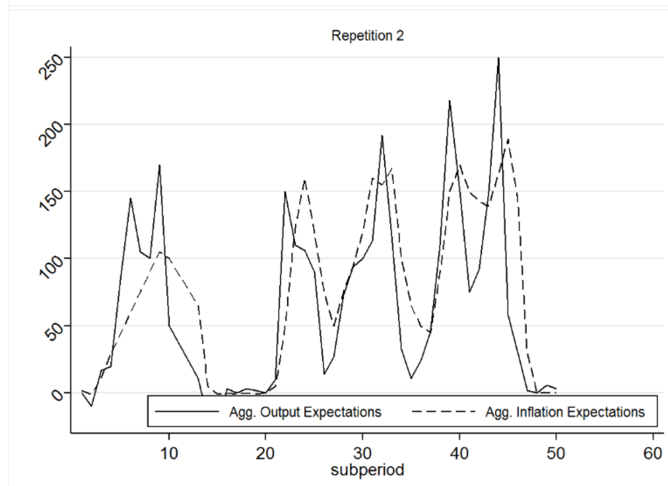




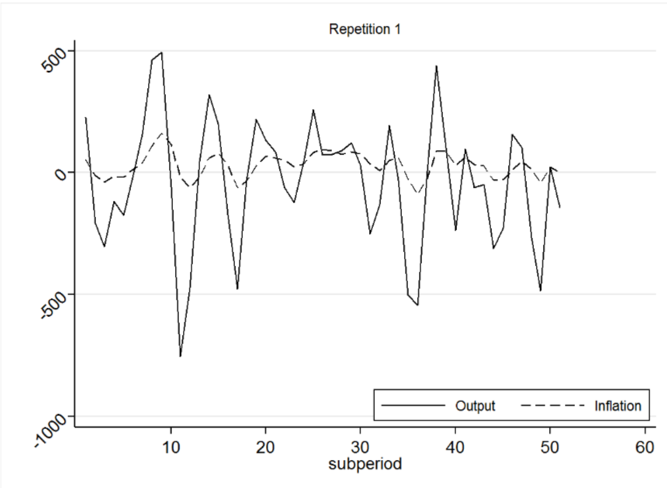


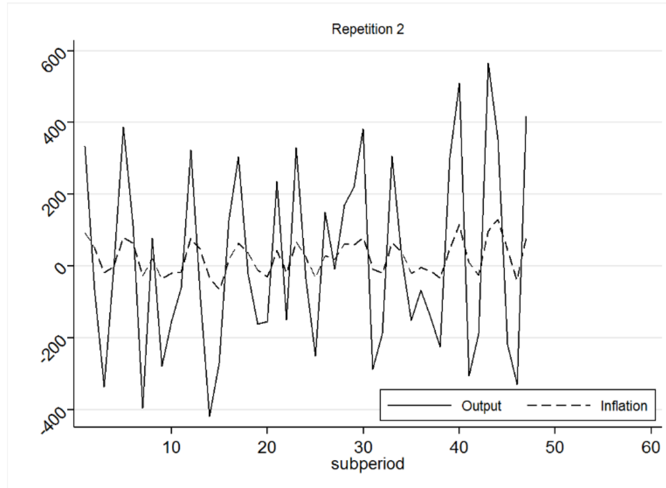


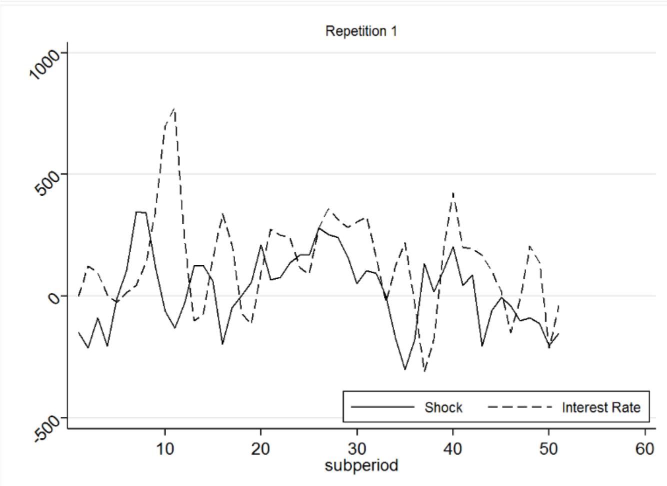


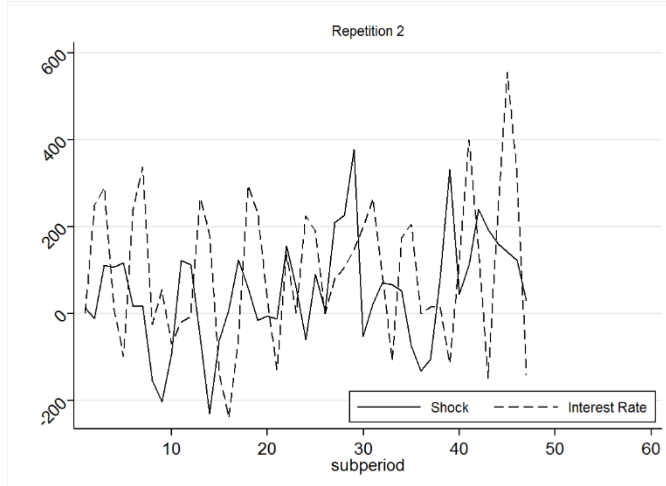


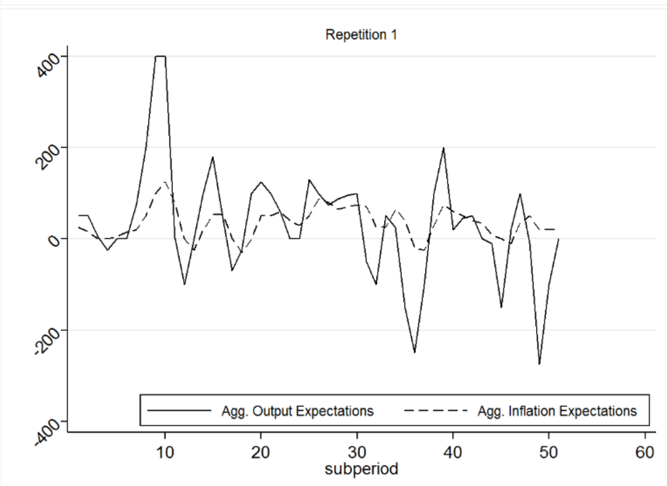
Aggressive Monetary Policy – Session 3

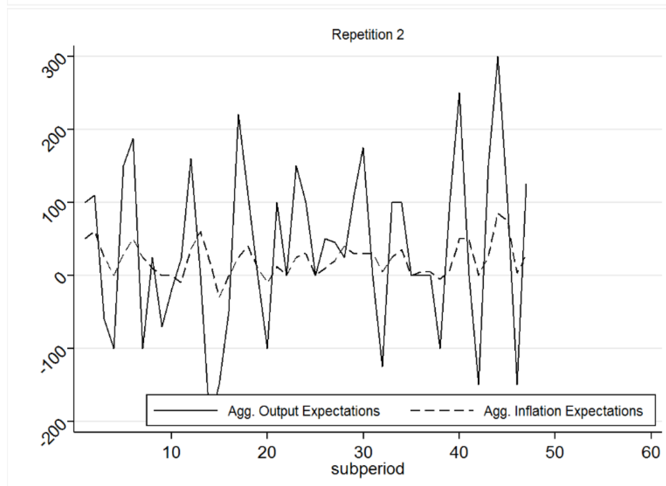




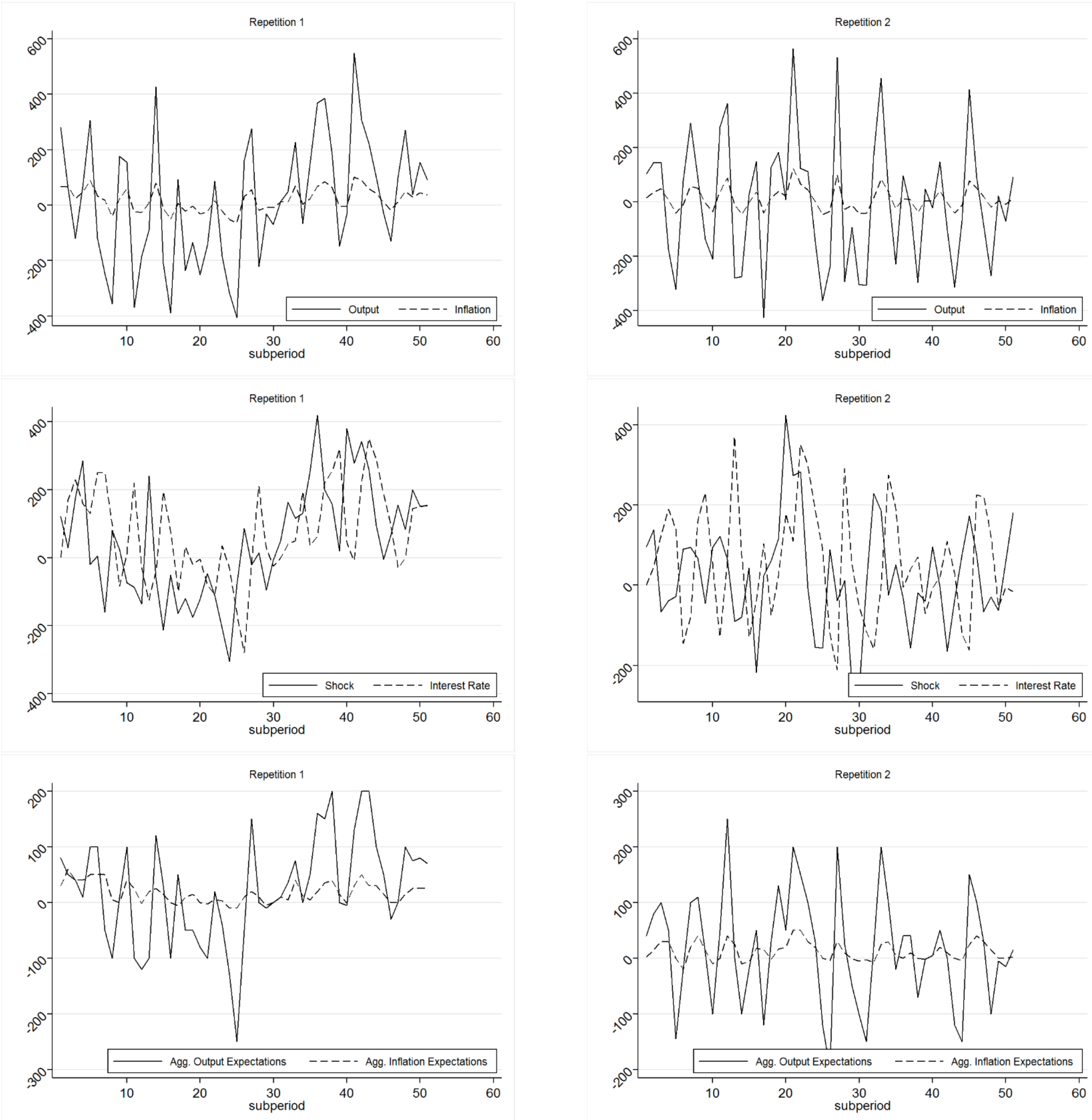




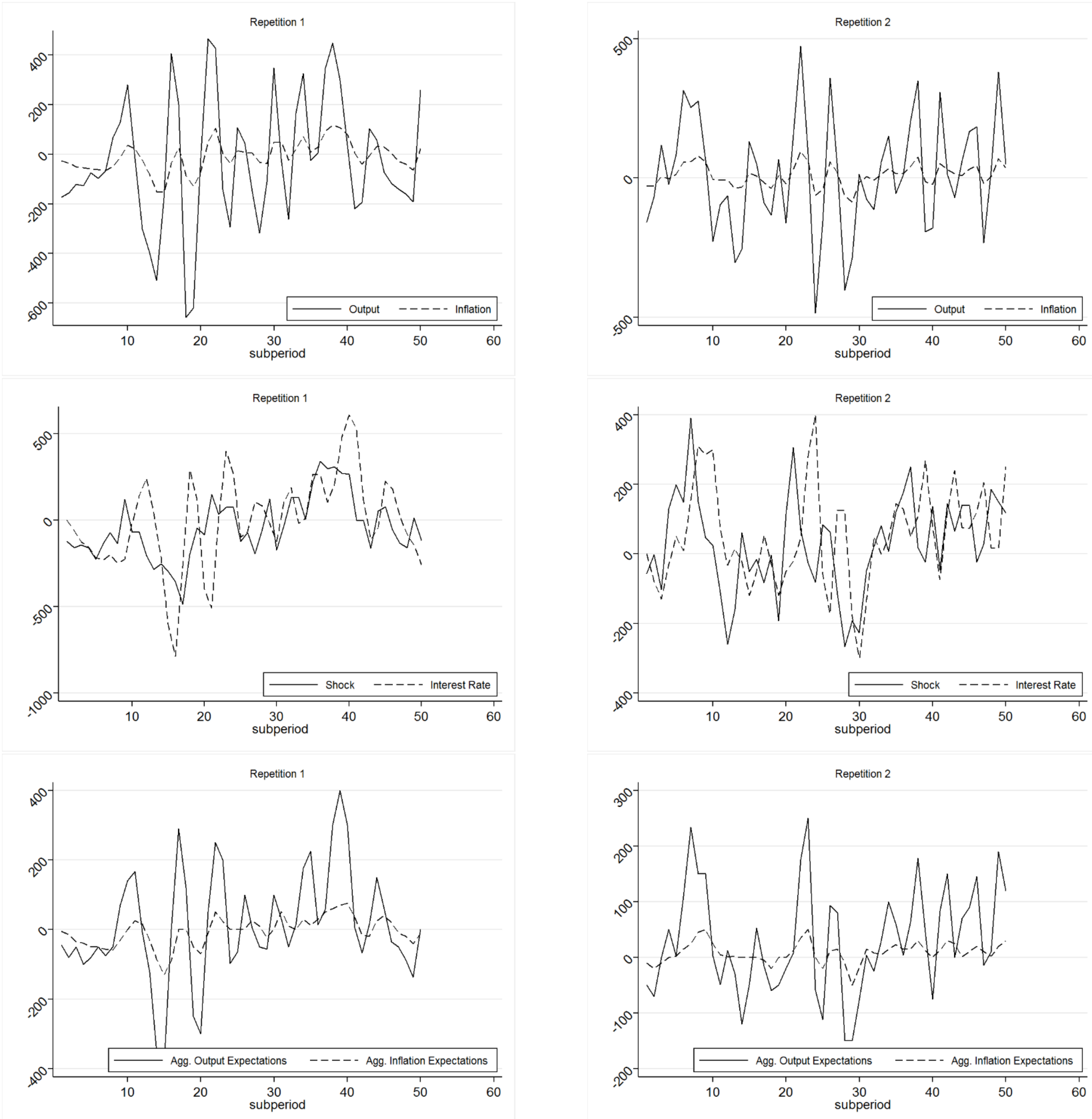




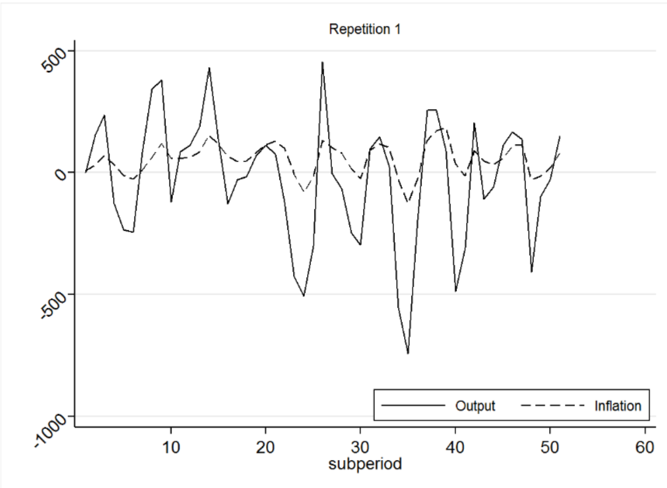
Aggressive Monetary Policy – Session 4

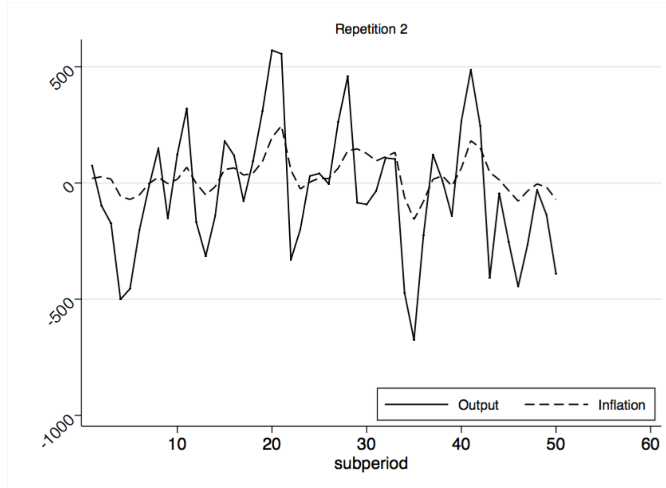


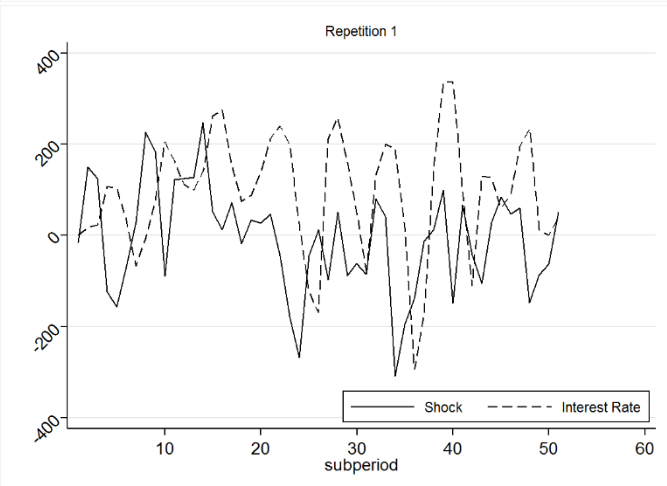
Aggressive Monetary Policy – Session 5

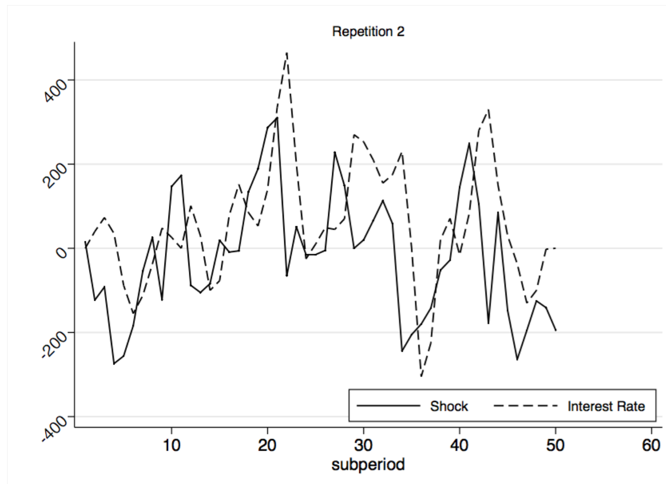


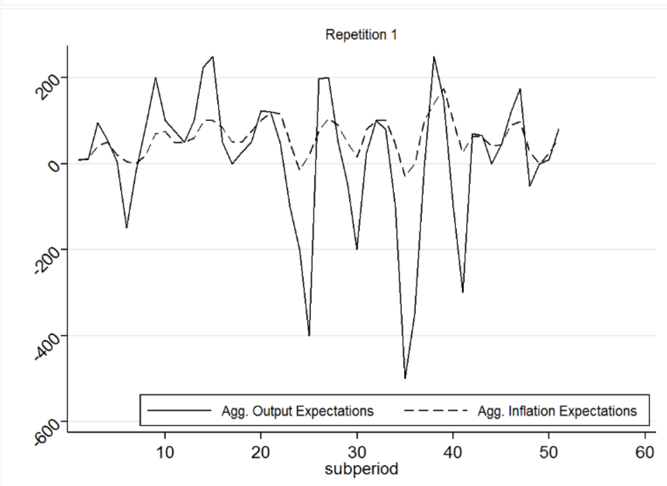
Communication – Session 1

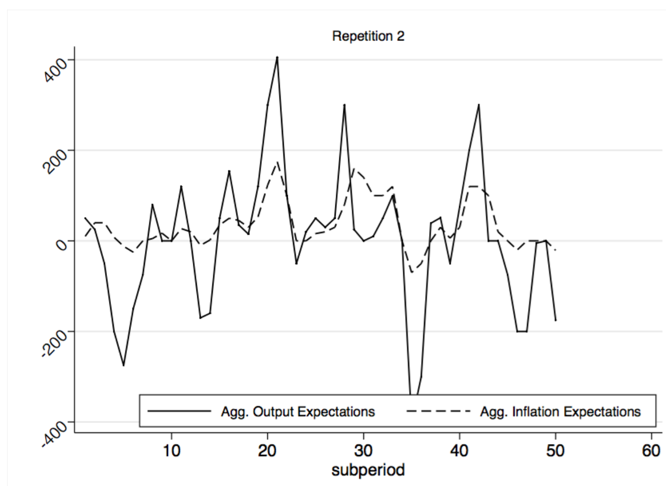




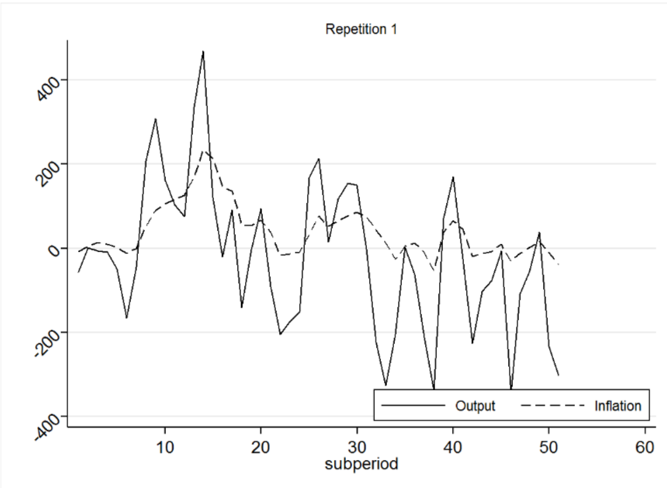


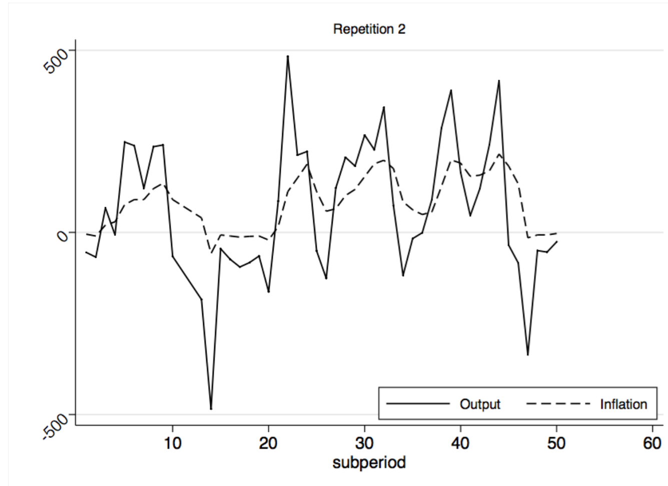


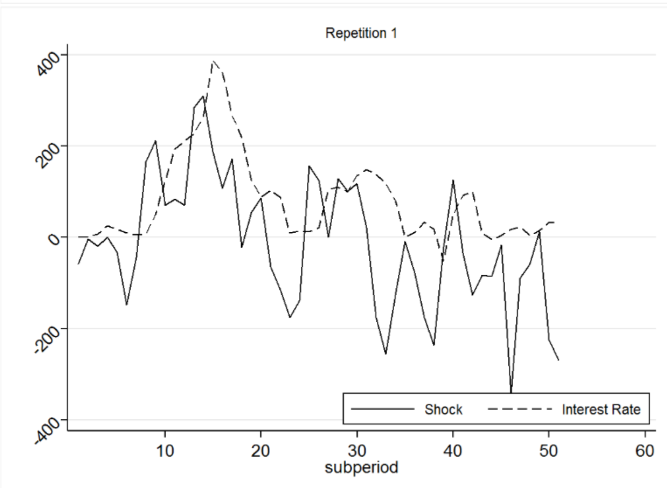


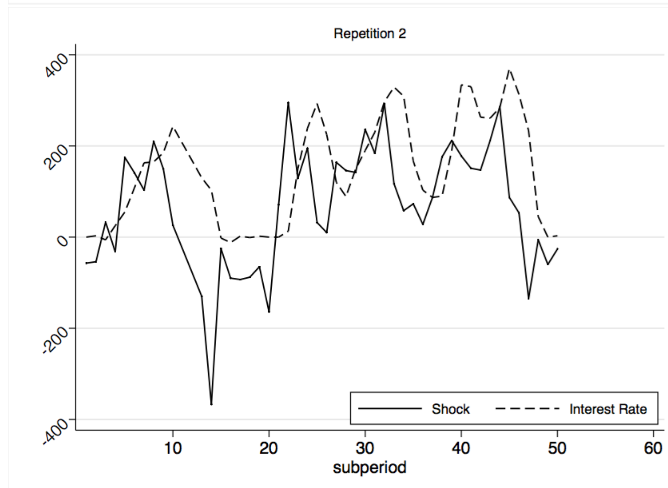


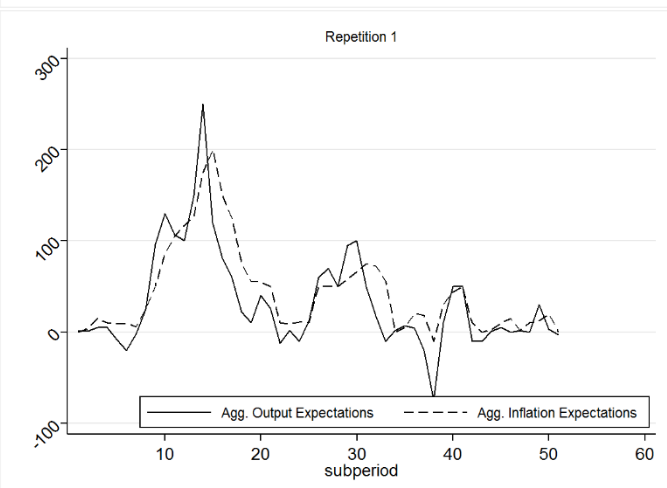
Communication – Session 2

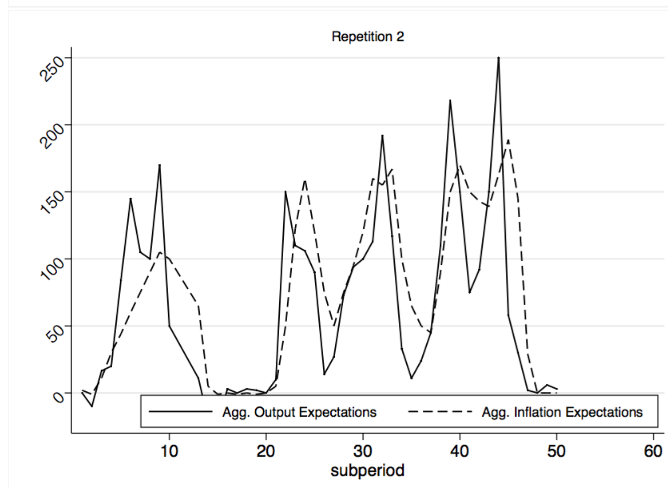




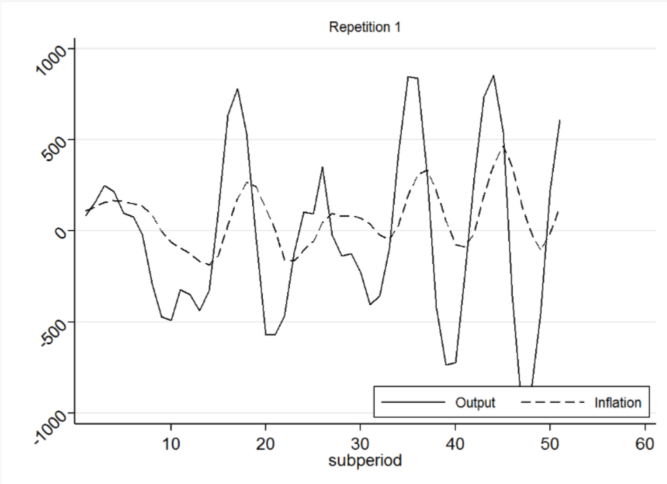


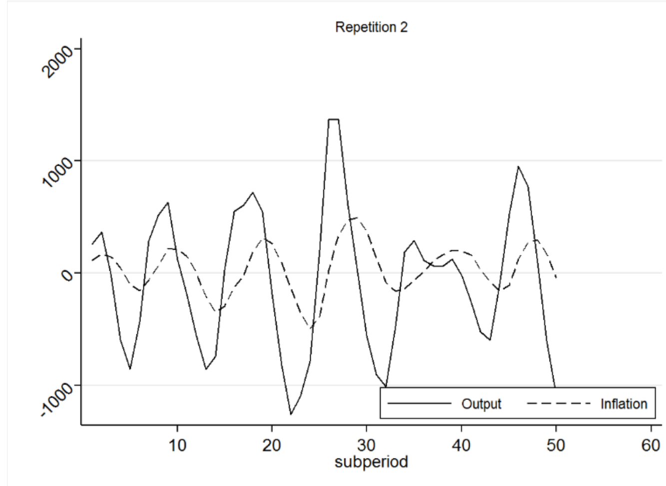


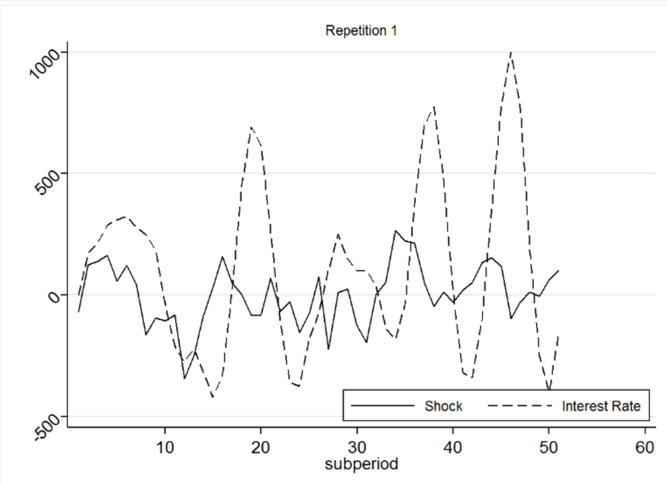


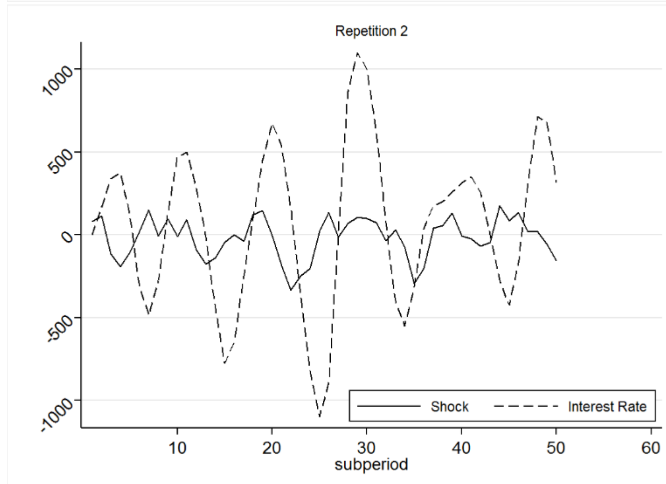


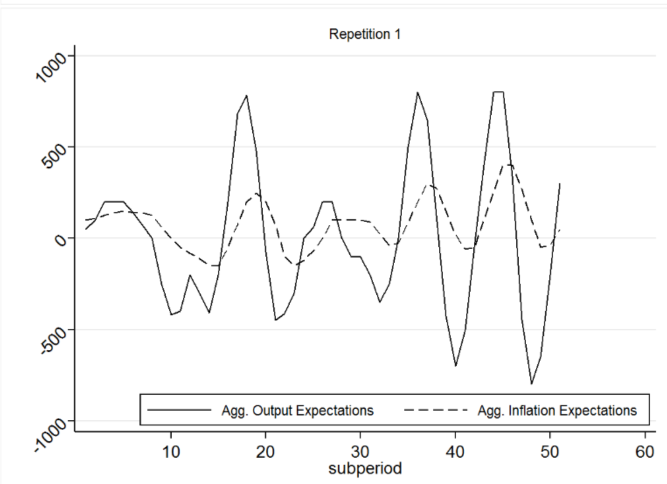
Communication – Session 3

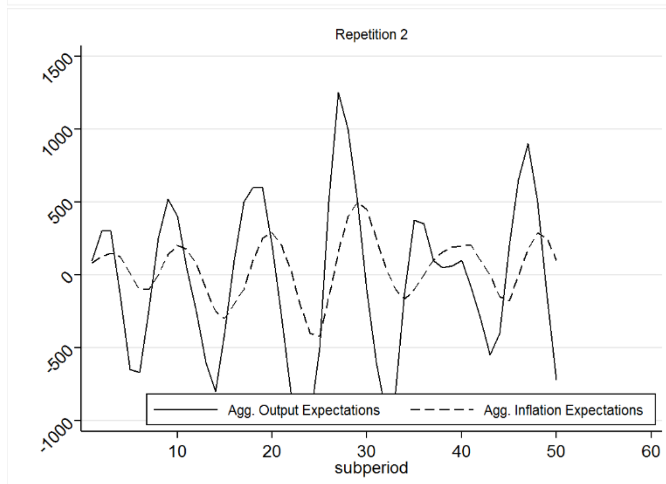




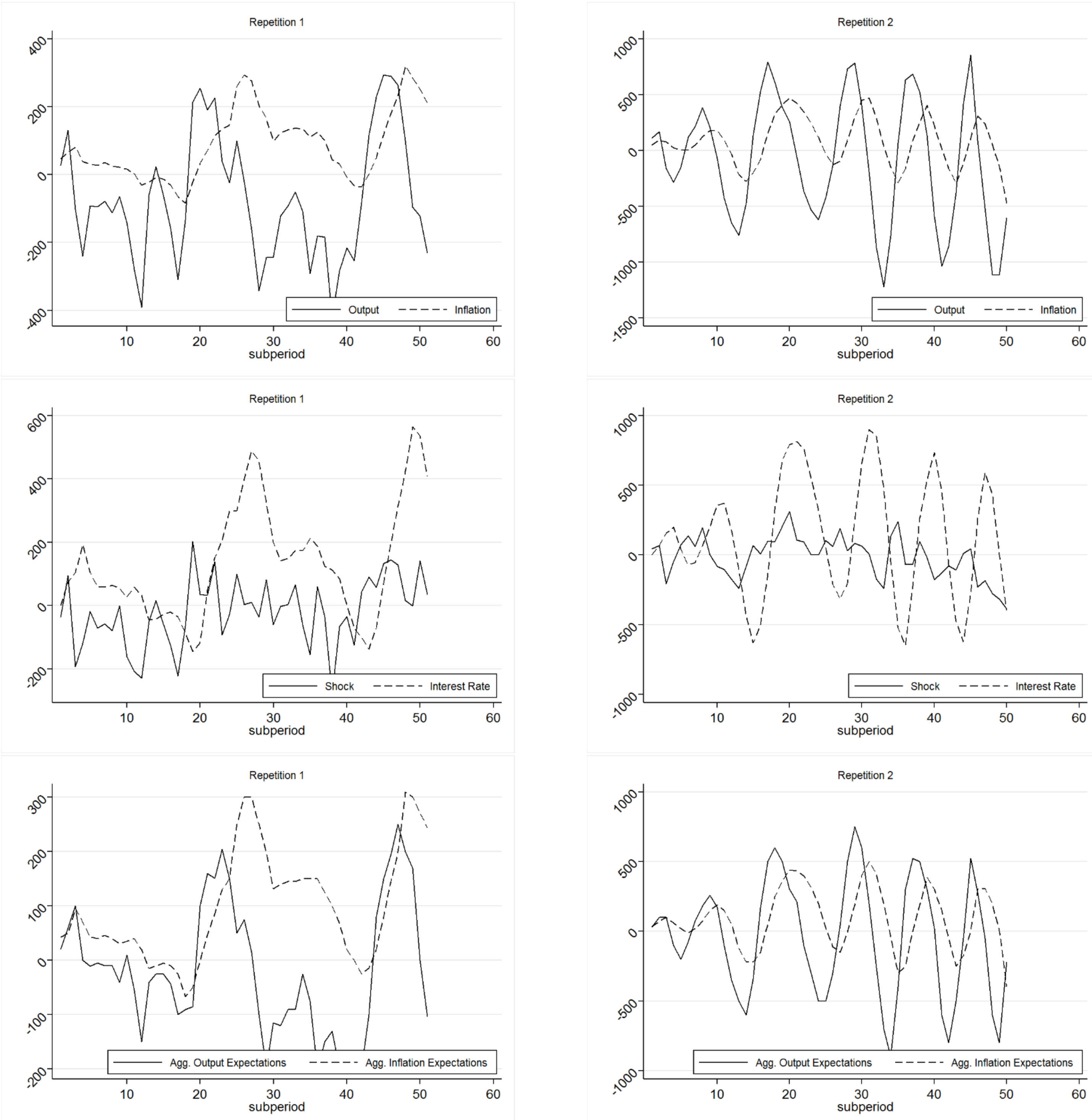




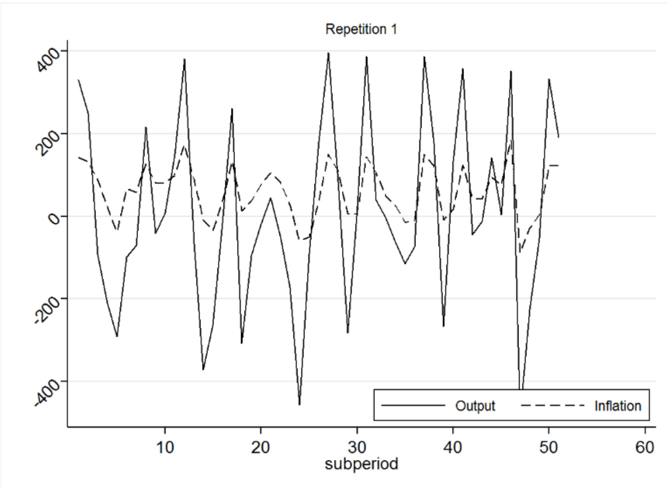


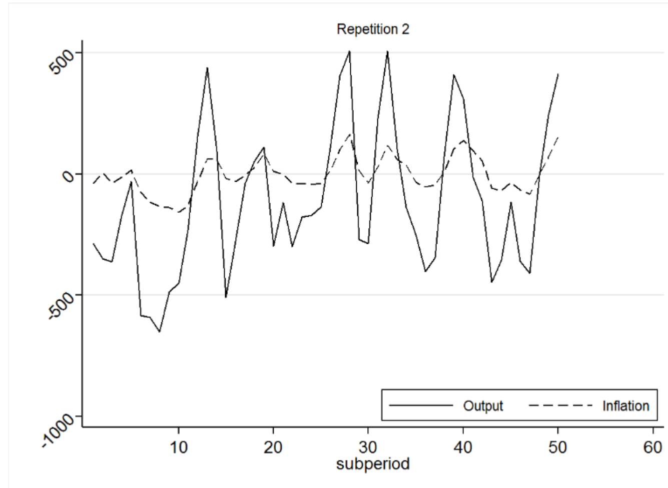


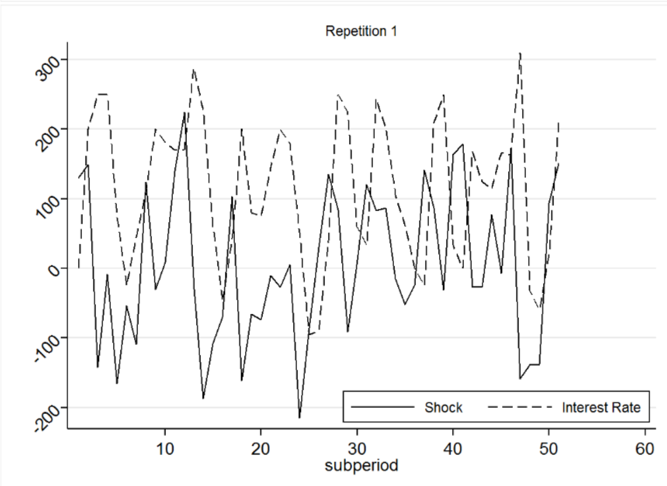
Communication – Session 4

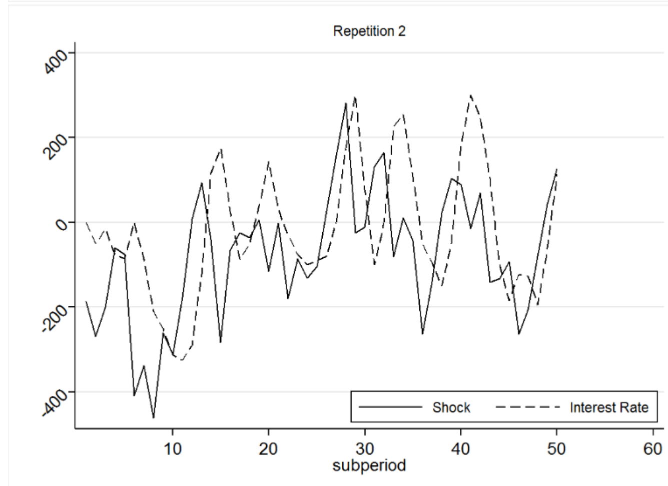


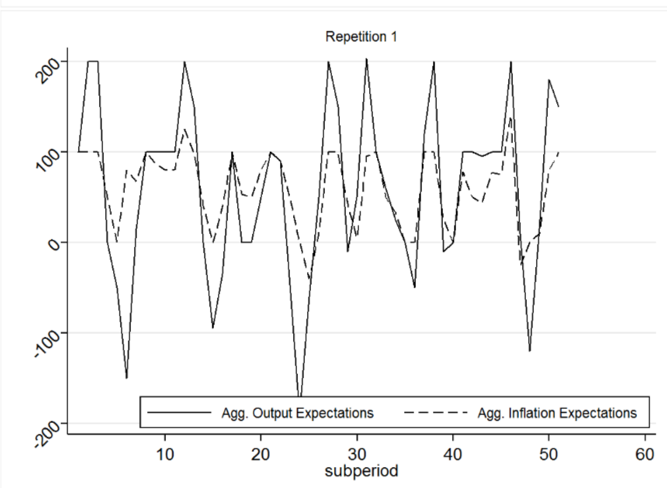
Communication – Session 5

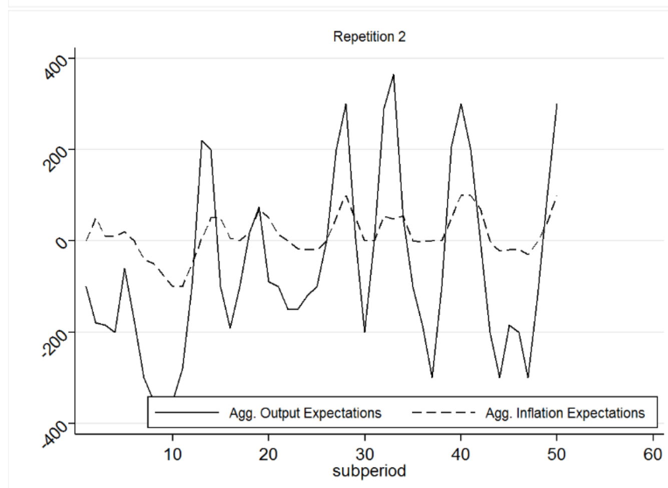












Communication – Session 6