ENSC-283

Assignment #1

Assignment date: Monday Jan. 12, 2009

Due date: Monday Jan. 19, 2009

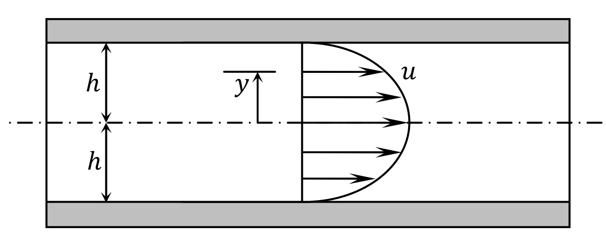
Problem: (Newtonian fluid shear stress)

The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates (see Figure) is given by the equation

$$u = \frac{3U_m}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$$

where U_m is the mean velocity. The fluid has the viscosity of $0.04\ lb.s/ft^2$. If $U_m = 2\ ft/s$ and $h = 0.2$ in, determine:

(a) The shearing stress acting on the bottom wall.
(b) The shearing stress acting on a plane parallel to the walls and passing through the centerline (midplane).



Solution

For this type of parallel flow the shearing stress is obtained from

$$\tau = \mu \frac{du}{dy} \quad (1)$$

Thus, if the velocity distribution $u = u(y)$ is known, the shearing stress can be determined at all points by evaluating the velocity gradient, du/dy. For the distribution given

$$\frac{du}{dy} = -\frac{3Uy}{h^2} \quad (2)$$

(a) The bottom wall $y = -h$ so that

$$\frac{du}{dy} = \frac{3U}{h} \quad (3)$$

And therefore the shearing stress is

$$\tau_{bottom\ wall} = \mu\left(\frac{3U}{h}\right) = \frac{\left(0.04\ lb.\frac{s}{ft^2}\right)(3)\left(2\frac{ft}{s}\right)}{(0.2\ in.)\left(1\frac{ft}{12in.}\right)} = 14.4\ lb/ft^2 (in\ direction\ of\ flow)$$

Always use units in your calculations

This stress creates a drag on the wall. Since the velocity distribution is symmetrical, the shearing stress along the upper wall would have the same magnitude and direction.

(b) Along the midplane where $y = 0$ it follows from Eq. (2) that

$$\frac{du}{dy} = 0 \quad (4)$$

And thus the shearing stress is

$$\tau_{midplane} = 0$$

Note (1):
In this problem, we calculated the shearing stress acting on the walls. Same shearing stress but in the opposite direction is acting on the fluid. Why?

Note (2):
From Eq. (2) we see that the velocity gradient is a linear function of y i.e. $\tau = \alpha y$ with $\alpha = -3U/h^2$. Hence, the shearing stress (see Eq. (1)) varies linearly with y and in this particular problem varies from 0 at the center of the channel to $14.4\ lb/ft^2$ at the walls. For the more general case, the actual variation will depend on the nature of the velocity distribution.