ENSC-283

Assignment #2

Assignment date: Monday Jan. 19, 2009

Due date: Monday Jan. 26, 2009

Problem1: (hydrostatic force on a plane circular surface)

The 4-m diameter circular gate of Figure 1 is located in the inclined wall of a large reservoir containing water ($\gamma = 9.80\ kN/m^3$). The gate is mounted on a shaft along its horizontal diameter, and the water depth is $10\ m$ above the shaft. Determine:

(a) The magnitude and location of the resultant force exerted on the gate by the water.

(b) The moment that would have to be applied to the shaft to open the gate.

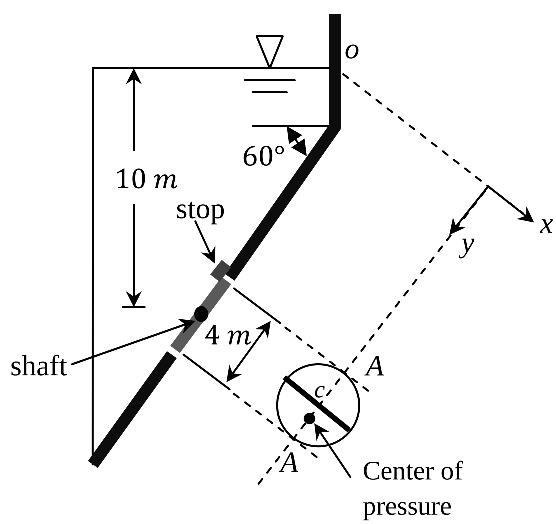

Figure 1 large reservoir of water

Solution

(a) To find the magnitude of the force of the water we can use the following equation

$$F_R = \gamma h_c A \tag{1}$$

and because the vertical distance from the fluid surface to the centroid of the area is 10 m it follows that

$$F_R = \left(9.81 \times 10^3 \left[\frac{N}{m^3}\right]\right)(10\ [m])(4\pi\ [m^2]) = 1.23\ MN$$

To locate the point (center of pressure) through which F_R acts, we use Eq. (2) and (3),

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \tag{2}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \tag{3}$$

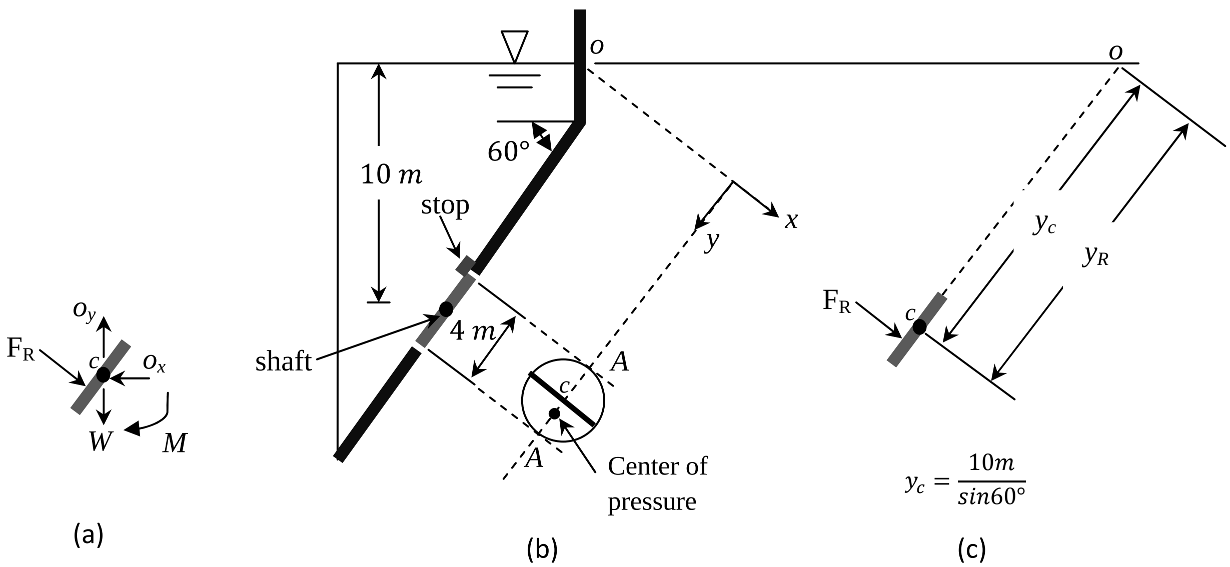

Figure 2 force diagram of the reservoir

For the coordinate system shown, $x_R = 0$ since the area is systematical, and the center of pressure must lie along the diameter $A - A$. To obtain y_R, we have from Fig. 2-13 of the text boSok,

$$I_{xc} = \frac{\pi R^4}{4} \tag{4}$$

and $y_c = 10/sin60°$ is shown in Fig. 2-c, thus,

$$y_R = \frac{(\pi/4)(2\ [m])^4}{(10\ [m]/sin\ 60°)(4\pi\ [m^2])} + \frac{10\ [m]}{sin60°} = 11.6\ [m]$$

The distance (along the gate) below the shaft to the center of pressure is then

$$y_R - y_c = 0.0866\ [m]$$

Note: Pressure force is always perpendicular to the gate surface.

(b) To calculate the moment required to open the gate, free body diagram shown in Fig. 2-a should be used. In this diagram, W is the weight of the gate and O_x and O_y are the horizontal and vertical reactions of the shaft on the gate. Since static condition is established

$$\sum M_c = 0 \tag{5}$$

And therefore,

$$M = F_R(y_R - y_c) = (1230 \times 10^3[N])(0.0866\ [m]) = 1.07 \times 10^5\ [N.M]$$

Problem2: (use of the pressure prism concept)

A pressurized tank contains oil ($SG = 0.90$) and has a square, $0.6\ m \times\ 0.6\ m$ plate bolted to its inside, as illustrated in Figure 2. The pressure gage on the top of the tank reads $50\ kPa$, and the outside of the tank is at atmospheric pressure. What is the magnitude and location of the resultant force on the attached plate?

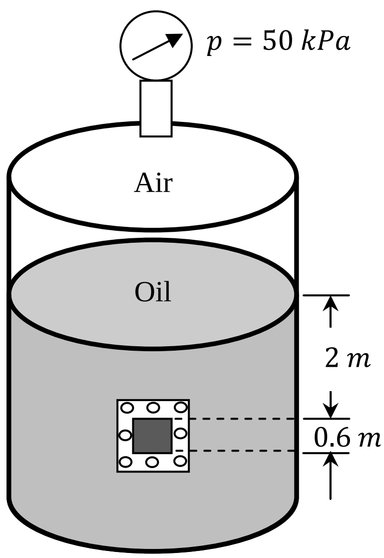

Figure 3 pressurized tank

Solution

The pressure distribution acting on the inside surface of the plate is shown in Fig. 3. The pressure at a given point on the plate is due the air pressure, p_s, at the oil surface, and the pressure due the oil which varies linearly with depth as is shown in Fig. 4. The resultant force on the plate (F_R) is due to the components, F_1 and F_2, where F_1 and F_2 are due to the rectangular and triangular portions of the pressure distribution, respectively. Thus,

$$F_1 = (p_s + \gamma_{oil} h_1)A$$
$$= \left(50 \times 10^3 \left[\frac{N}{m^2}\right] + 0.9 \times\ 9.81 \times 10^3\ \left[\frac{N}{m^3}\right] \times 2\ [m]\right)(0.36\ [m^2])$$
$$= 24.4 \times 10^3 [N]$$

and

$$F_2 = \gamma_{oil}\left(\frac{h_2 - h_1}{2}\right)A = \left(0.9 \times\ 9.81 \times 10^3\ \left[\frac{N}{m^3}\right]\right)\left(\frac{0.6\ [m]}{2}\right)(0.36\ [m^2])$$
$$= 0.954 \times 10^3 [N]$$

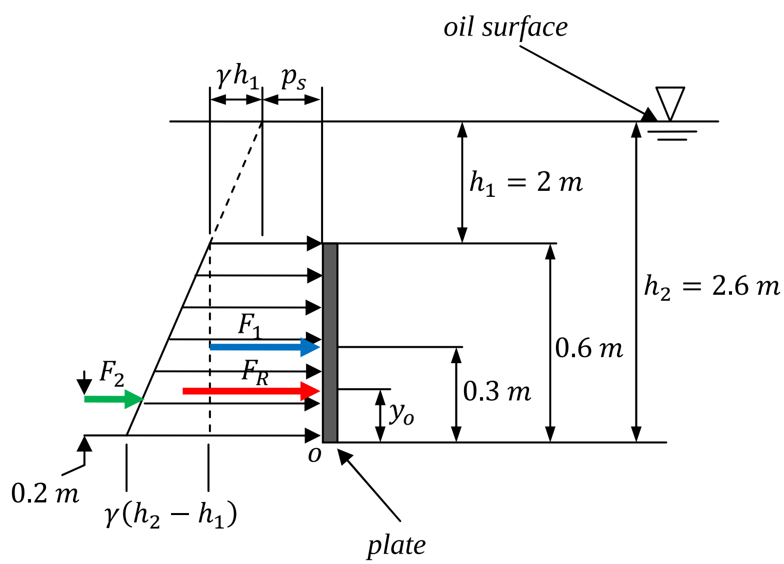

Figure 4 force diagram of the pressurized tank

The magnitude of the resultant force, F_R, is therefore,

$$F_R = F_1 + F_2 = 25.4\ [kN]$$

The vertical location of F_R can be obtained by summing moments around an axis through point o. Thus,

$$F_R y_o = F_1(0.3\ [m]) + F_2(0.2\ [m])$$

$$y_o = \frac{(24.4\times10^3\ [N])(0.3\ [m]) + (0.954\times10^3 [N])(0.2\ [m])}{24.4\times10^3\ [N]} = 0.296\ [m]$$

Note: The air pressure used in the calculation of the force was gage pressure. Atmospheric pressure does not affect the resultant force, as it acts on both sides of the plate, therby canceling its effect.