ENSC-283

Assignment #3

Assignment date: Monday Jan. 26 2009

Due date: Monday Feb. 2, 2009

Problem1: (force components on a curved submerged surface)

The gate shown in Fig. 1 is hinged at o and has constant width, $w = 5\ m$. The equation of the surface is $x = y^2/a$, where $a = 4\ m$. The depth of water to the right of the gate is $D = 4\ m$. Find the magnitude of force, F_a, applied as shown, required to maintain the gate in equilibrium if the weight of the gate is neglected.

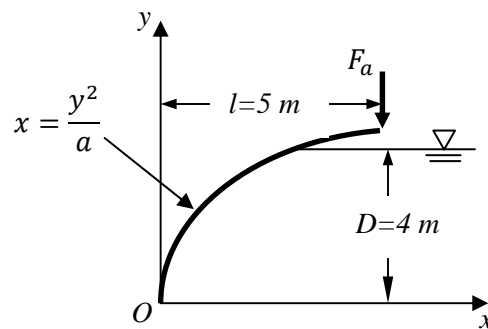**Figure 1 curved gate**

Solution

Our approach to solve this problem is to:

1- Draw a free body diagram (FBD) for the system.
2- Determine the magnitude of vertical and horizontal forces, F_H and F_V.
3- Find the point of action of each component.
4- Take moments about the origin, i.e. point O.

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Solution trick: We know that the vertical component of the fluid force is equal to the weight of fluid (in magnitude and location) above the curved surface. However, there is no fluid above the gate surface. We know that there is vertical component acting on the surface of the gate and it is upward. **(Why?)** How we can find this vertical component? Let's consider Fig. (S1-a). What is special about this case?
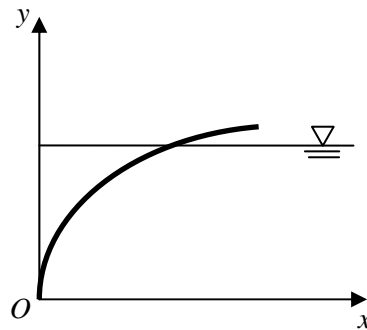

Figure S1-a Null fluid forces

Due to same pressure on both sides of the surface, vertical and horizontal forces acting on both sides of the gate are in balance. Now let's consider Fig. (S1-b).
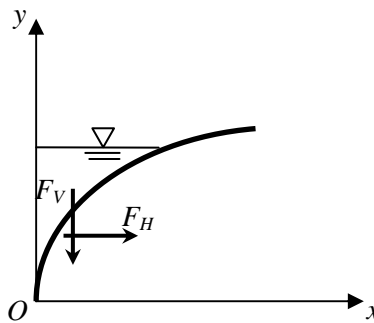

Figure S1-b Fluid on the top of the surface

You can now easily find the vertical and horizontal forces acting on the gate. These the two forces we are looking for but in the opposite direction. **(Why? Look at the FBD of the actual problem, i.e. Fig. S1-c)**
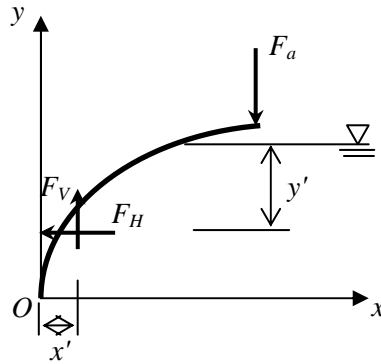
Figure S1-c Free body diagram of actual problem

To take a moment about point O, we need to know the magnitude of F_H and F_V forces as well as x' and y'.

Horizontal force, F_H:

Magnitude of the horizontal force is given by the magnitude of the force on an equivalent projected vertical flat plate:

$$F_H = p_c A = \gamma h_c A$$

where $h_c = D/2$ and $A = D \times w$. Thus

$$F_H = \gamma \frac{D^2}{2} w = 9810 \left[\frac{N}{m^3}\right] \times \frac{(4\,[m])^2}{2} \times 5[m] = 392\,kN$$

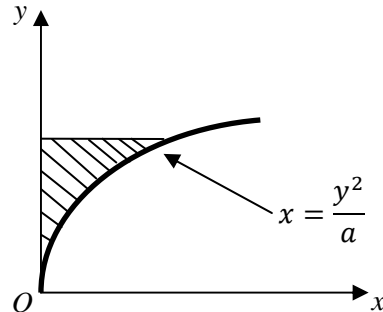Point of action for the horizontal force can be obtained from

$$y' = y_c + \frac{I_{xx}}{Ay_c}$$

where $y_c = h_c$ and $I_{xx} = wD^3/12$ for rectangular surface. Hence

$$y' = \frac{D}{2} + \frac{wD^3/12}{w \times D \times D/2} = \frac{2D}{3} = \frac{2 \times 4[m]}{3} = 2.67\,[m]$$

Vertical force, F_H:

To calculate the vertical force, one should compute the weight of water above the gate as we discussed before. To do so, area of the dashed zone in the following figure times the width of the gate should be calculated.

Total volume of the fluid above the gate is

$$\forall = \int_0^D \frac{y^2}{a} w dy = \frac{wD^3}{3a}$$

Hence vertical force can be obtained from

$$F_V = \gamma \forall = \gamma \frac{wD^3}{3a} = \frac{9810[N/m^3] \times 5[m] \times (4[m])^3}{3 \times 4[m]} = 261 \ kN$$

To determine the point of action for the vertical force, you should first find the center of gravity for the water above the gate. Center of gravity can be obtained if one notes that the moment of F_V and the moment of the sum of the differential weights about y axis must be equal (See the following Figure). so

$$x'F_V = \gamma \int_0^{D^2/a} x(D-y)w dx$$

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x
$\overleftrightarrow{x'}$
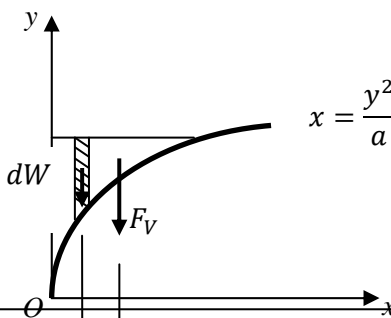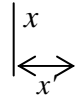
Substituting $y = \sqrt{ax}$,

$$x'F_V = \gamma \int_0^{D^2/a} x(D - \sqrt{ax})wdx = \frac{\gamma wD^5}{10a^2}$$

Hence

$$x' = \frac{\gamma wD^5}{10a^2}\frac{1}{F_V} = \frac{\gamma wD^5}{10a^2} \times \frac{3a}{\gamma wD^3} = \frac{3D^2}{10a} = \frac{3 \times (4[m])^2}{10 \times 4[m]} = 1.2\ [m]$$

The final step to determine F_a is to balace the moments about point O

$$\sum M_o = -lF_a + x'F_V + (D - y')F_H = 0$$

Therefore

$$F_a = \frac{[x'F_V + (D - y')F_H]}{l}$$
$$= \frac{1}{5[m]}\{1.2[m] \times 261[kN] + (4 - 2.67)[m] \times 392[kN]\} = 167\ kN$$

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Problem2: (free surface shape of liquid in a rotating tank)

It has been suggested that the angular velocity, ω, of a rotating body or shaft can be measured by attaching an open cylinder of liquid, as shown in Fig. 2, and measuring with some type of depth gage, the change in the fluid level, $H - h_0$, caused by the rotation of the fluid. Determine the relationship between this change in fluid level and the angular velocity.
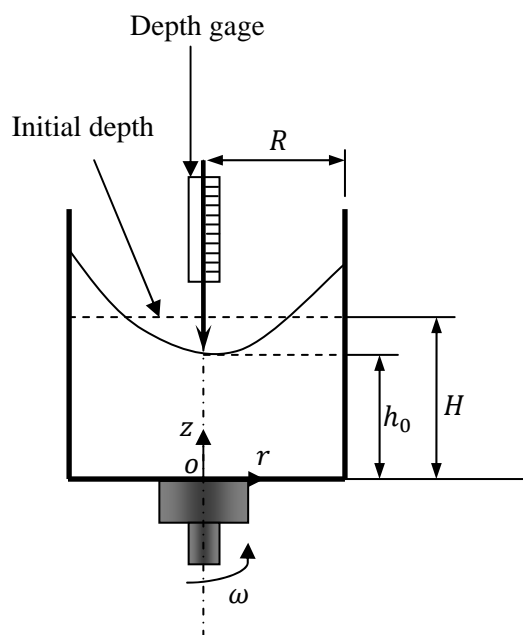

Figure 2 rotating body

Solution

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The height of the free surface, h, above the tank can be determined from the following equation

$$h = \frac{\omega^2 r^2}{2g} + h_2 \qquad (1)$$

The initial volume of the fluid in the tank is

$$\forall_i = \pi R^2 H \qquad (2)$$

The volume of the fluid in the rotating tank can be found

$$\forall = 2\pi \int_0^R r \left(\frac{\omega^2 r^2}{2g} + h_2 \right) dr \qquad (3)$$

Letting $\forall = \forall_i$ and using Eqs. (2) and (3)

$$\pi R^2 H = \frac{\pi \omega^2 R^4}{4g} + \pi R^2 h_0$$

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