ENSC-283

Assignment #4

Assignment date: Monday Feb. 2, 2009

Due date: Monday Feb. 9, 2009

Problem1: (Nozzle)

The converging-diverging nozzle shown in Fig. 1 expands and accelerates dry air to supersonic speeds at the exit, where $p_2 = 8\,kPa$ and $T_2 = 240\,K$. At the throat, $p_1 = 284\,kPa, T_1 = 665\,K$, and $V_1 = 517\,m/s$. For steady compressible flow of an ideal gas, estimate (a) the mass flow in kg/h, (b) the velocity V_2, and (c) the Mach number Ma_2.

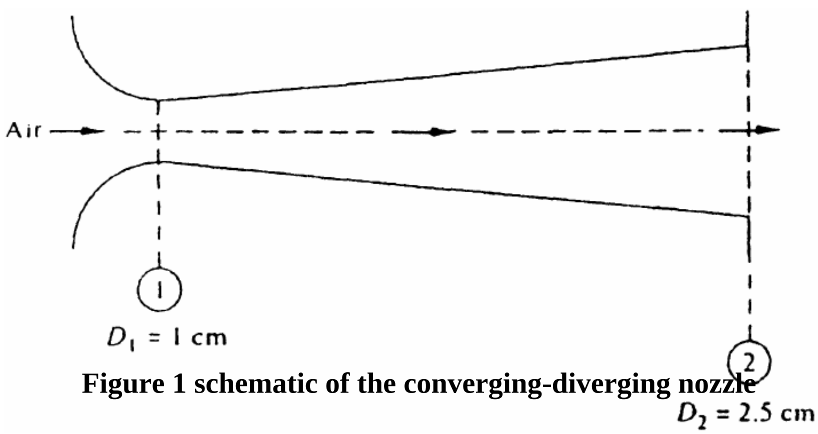

Figure 1 schematic of the converging-diverging nozzle

Solution

The mass flow is given by the throat conditions:

$$\dot{m} = \rho_1 A_1 V_1$$

where A_1 is the cross-sectional area at point 1. Density of the gas can be obtained from the following equation

$$\rho = \frac{p}{RT}$$

where $R = 287\ J/K.kg$ for air and T is in Kelvin. Hence, for point 1,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{284000\ [pa]}{287\ \left[\frac{J}{kg.K}\right] \times 665\ [K]} = 1.3\ \left[\frac{kg}{m^3}\right]$$

and mass flow rate is

$$\dot{m}_1 = 1.3\ \left[\frac{kg}{m^3}\right] \times \left(\pi \frac{0.01^2}{4} [m^2]\right) \times 517 \left[\frac{m}{s}\right] = 0.0604\ \left[\frac{kg}{s}\right]$$

For steady flow, mass flow rate remains constant, hence,

$$\dot{m}_1 = \dot{m}_2 = \rho_2 A_2 V_2$$

therefore

$$V_2 = \frac{\dot{m}_1}{\rho_2 A_2} = \frac{\dot{m}_1}{\frac{p_2}{RT_2} A_2} = \frac{0.0604\ \left[\frac{kg}{s}\right]}{\frac{8000[pa]}{287\left[\frac{J}{kg.K}\right] \times 240[K]} \times \left(\pi \frac{0.025^2}{4} [m^2]\right)} = 1060 \left[\frac{m}{s}\right]$$

Recall that the speed of sound of an ideal gas is $a = \sqrt{kRT}$ where $k = c_p/c_v$ and Mach number is defined as (what is the unit of Mach number?)

$$Ma = \frac{V}{a}$$

At point 2 the Mach number is

$$Ma_2 = \frac{V_2}{a} = \frac{1060\left[\frac{m}{s}\right]}{\sqrt{1.4 \times 287\left[\frac{J}{kg.K}\right] \times 240[K]}} = 3.14$$

Problem 2 (water jet)

A steady two-dimensional water jet, $4\,cm$ thick with a weight flow rate of $1960\,N/s$, strikes an angled barrier as in Fig. 2. Pressure and water velocity are constant everywhere. Thirty percent of the jet passes through the slot. The rest splits symmetrically along the barrier. Calculate the horizontal force F needed, per unit thickness into the paper, to hold the barrier stationary.

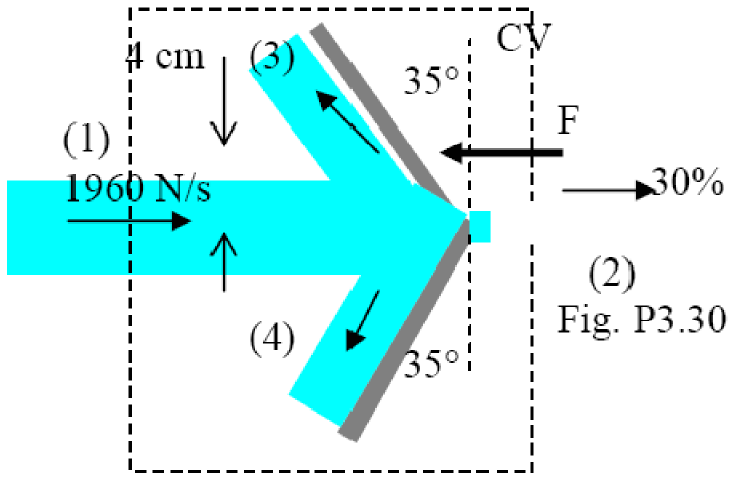

Figure 2 schematic of water jet

Solution

For water take $\rho = 998\,kg/m^3$. The control volume (see figure) cuts through all four jets, which are numbered. The velocity of all jets follows from the weight flow at (1)

$$V_1 = \frac{\dot{w}}{\rho g A_1} = \frac{1960\left[\frac{N}{s}\right]}{9.81\left[\frac{m}{s^2}\right] \times 998\left[\frac{kg}{m^3}\right] \times 0.04[m] \times 1[m]} = 5\left[\frac{m}{s}\right]$$

$$\dot{m}_1 = \frac{\dot{w}}{g} = \frac{1960\ \left[\frac{N}{s}\right]}{9.81\ \left[\frac{m}{s^2}\right]} = 200\left[\frac{kg}{s.m}\right]$$

$$\dot{m}_2 = 0.3\dot{m}_1 = 60\left[\frac{kg}{s.m}\right]$$

$$\dot{m}_3 = \dot{m}_4 = \frac{\dot{m}_1 - \dot{m}_2}{2} = 0.35\dot{m}_1 = 70\left[\frac{kg}{s.m}\right]$$

Then the x-momentum relation for this control volume yields

$$\sum F_x = -F = \dot{m}_2 V_2 + \dot{m}_3 V_3 + \dot{m}_4 V_4 - \dot{m}_1 V_1$$

$$-F = 60\left[\frac{kg}{s.m}\right] \times 5\ \left[\frac{m}{s}\right] + 70\left[\frac{kg}{s.m}\right] \times \left(-5\ \left[\frac{m}{s}\right] \times cos55°\right) + 70\left[\frac{kg}{s.m}\right] \times \left(-5\ \left[\frac{m}{s}\right] \times cos55°\right) - 200\left[\frac{kg}{s.m}\right] \times 5\ \left[\frac{m}{s}\right] = 1100\left[\frac{N}{m}\right]$$