ENSC-283

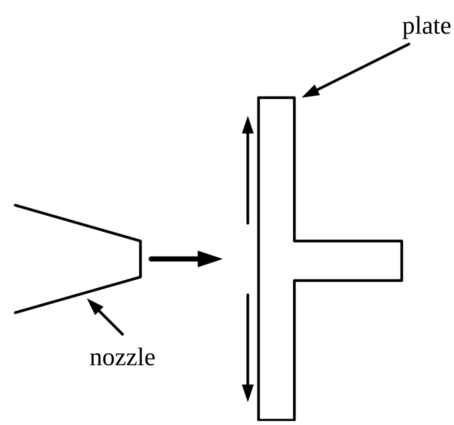
Assignment #5

Assignment date: Monday Feb. 9, 2009

Due date: Monday Feb. 16, 2009

Problem1: (choice of control volume for momentum analysis)

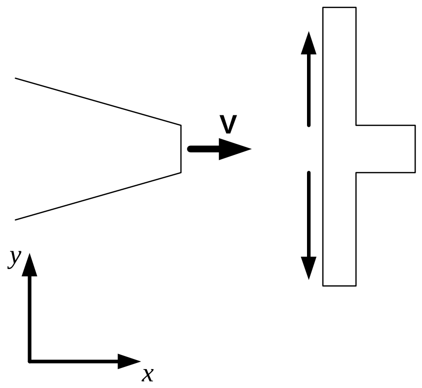
Water from a stationary nozzle strikes a flat plate as shown. The water leaves the nozzle at $15\ m/s$; the nozzle area is $0.01\ m^2$. Assuming the water is directed normal to the plate, and flows along the plate, determine the horizontal force you need to resist to hold it.



Solution:

Assumptions of this problem are:

- Steady flow
- Incompressible flow
- Uniform flow at each section where fluid crosses the CV boundaries



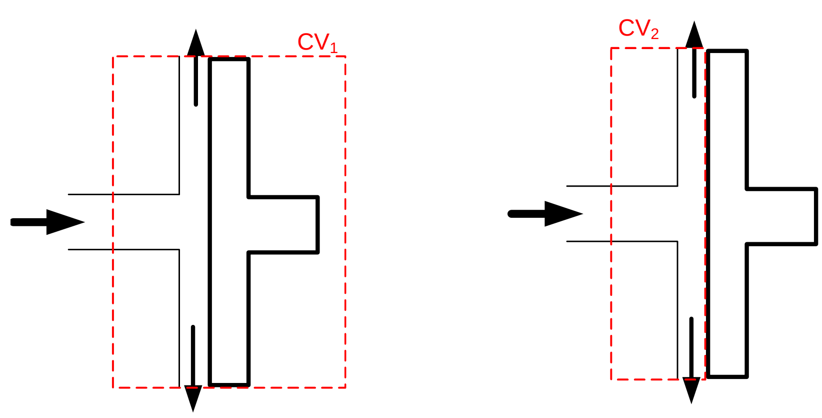
We chose a coordinate system in defining the problem above. We should choose a suitable control volume. Regardless of our choice of control volume, and our assumptions lead to

$$\boldsymbol{F} = \boldsymbol{F_S} + \boldsymbol{F_B} = \sum_{CS} \boldsymbol{V} \rho\, \boldsymbol{V}.\boldsymbol{A}$$

and

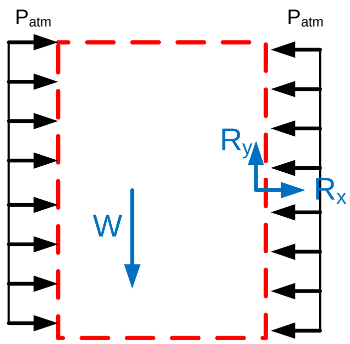
$$\sum_{CS} \rho\, \boldsymbol{V}.\boldsymbol{A} = 0$$

where F_B is body force. Two possible control volumes are shown by the dashed line below.



Evaluating the momentum flux term will lead to the same result for both control volumes.
Problem is solved for CV_1 and here and we let you solve the problem for CV_2.
The control volume has been selected so that the left surface is equal to the area of the right surface. Denote this area by A.

The control volume cuts through your hand. We denote the components of the reaction force of your hand on the control volume as R_x and R_y and assume both to be positive. Atmospheric pressure acts on all surfaces of the control volume. Note that the pressure in a free jet is ambient.



Body force in the *y* direction is denoted as W. Since we are looking for horizontal forces, we write the x component of the steady flow momentum equation

$$F = F_{S,x} + F_{B,x} = \sum_{CS} u\rho\, \boldsymbol{V}.\boldsymbol{A}$$

There are no body forces in the *x* direction, hence, $F_{B,x} = 0$.
To evaluate $F_{S,x}$, we must include all surface sources acting on the control volume:

$$F_{S,x} = P_{atm}A + R_x - P_{atm}A$$

Consequently, $F_{S,x} = R_x$.
On the other hand,

$$\sum_{CS} u\rho\ \boldsymbol{V}.\boldsymbol{A} = V\rho VA = -\rho V^2 A$$

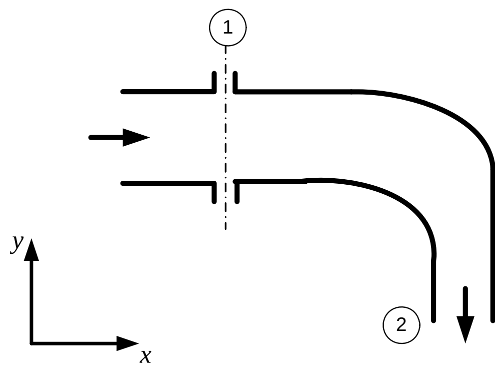
Note that jet velocity is in x direction so, $\boldsymbol{V}.\boldsymbol{A} = -VA$. Thus,

$$R_x = -999\left[\frac{kg}{m^3}\right] \times \left(15\left[\frac{m}{s}\right]\right)^2 \times 0.01[m^2] = -2.25\ kN$$

Note: As you see in this problem, since atmospheric pressure is acting on both sides of the control volume, it is canceled out.

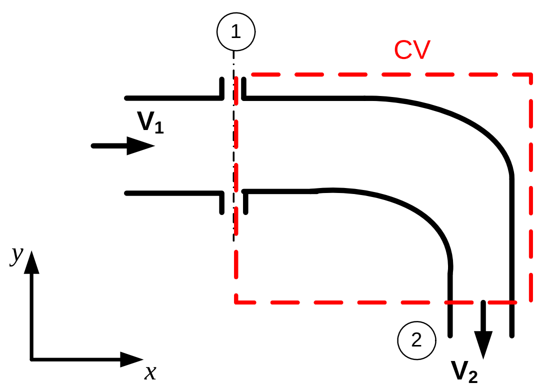
Problem 2 (use of gauge pressures)

Water flows steadily through the 90° reducing elbow shown in the diagram. At the inlet to the elbow, the absolute pressure is $220\ kPa$ and the cross-sectional area is $0.01\ m^2$. At the outlet, the cross-sectional area is $0.0025\ m^2$ and the velocity is $16\ m/s$. The elbow discharges to the atmosphere. Determine the force required to hold the elbow in place.

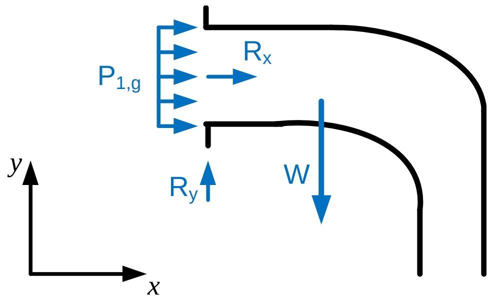


Solution:

Let us choose a control volume as shown.



Note that we have several surface forces: P_1 on area A_1 at point 1 and atmospheric pressure, P_{atm} everywhere else. The exit section 2, is a free jet and so pressure is ambient pressure. We can simplify this problem by subtracting P_{atm} from the entire surface. Then, final results will be in gage pressure. Force diagram of the elbow is shown as below



Our assumptions to solve this problem are:

1. Uniform flow at each section
2. Incompressible flow
3. Steady flow
4. Neglect the weight of elbow and water in the elbow

Since there is no body force in *x* direction, $F_{B,x} = 0$. All surface forces acting on the control volume surface in the *x* direction are:

$$F_{S,x} = P_{1,g}A_1 + R_x$$

This force in static condition should be balanced by momentum change in that direction, hence

$$F_{S,x} = \sum_{CS} u\rho\, \boldsymbol{V}.\boldsymbol{A} = V_1(-\rho V_1 A_1) = -\rho V_1^2 A_1$$

or

$$R_x = -P_{1,g}A_1 - \rho V_1^2 A_1$$

Note that the *x* component of the velocity at point 1 is V_1. To find V_1, mass conservation should be used

$$\rho V_1 A_1 = \rho V_2 A_2 \rightarrow V_1 = V_2 \frac{A_2}{A_1} = 16 \left[\frac{m}{s}\right] \times \frac{0.0025[m^2]}{0.01[m^2]} = 4 \left[\frac{m}{s}\right]$$

Therefore

$$R_x = -1.19 \times 10^5 \left[\frac{N}{m^2}\right] \times 0.01[m^2] - 999 \left[\frac{kg}{m^3}\right] \times \left(4 \left[\frac{m}{s}\right]\right)^2 \times 0.01\,[m^2]$$
$$= -1.35\,kN$$

Writing the y momentum equation gives

$$F_{S,y} + F_{B,y} = \sum_{CS} v\rho\, \boldsymbol{V}.\boldsymbol{A}$$

The only body force acting in y direction is the elbow weight and water weight. Since we assumed that these weights are negligible $F_{B,y} \approx 0$. Similar to the calculation of R_x,

$$R_y = -\rho V_2^2 A_2$$

Note that there is no pressure force in y direction. Substituting the values

$$R_y = -999 \left[\frac{kg}{m^3}\right] \times \left(16 \left[\frac{m}{s}\right]\right)^2 \times 0.0025\ [m^2] = -0.639\ kN$$