ENSC-283

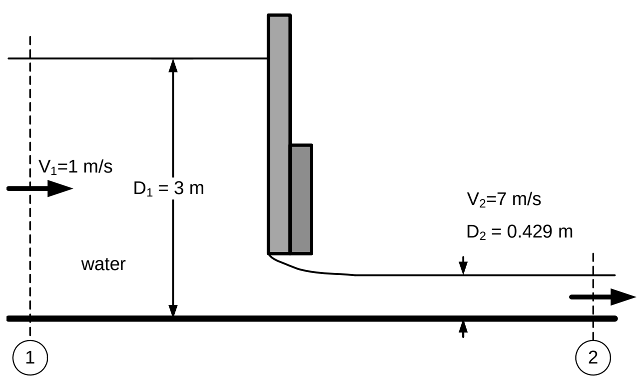
Assignment #6

Assignment date: Monday Feb. 23, 2009

Due date: Monday Mar. 2, 2009

Problem: (Flow under a sluice gate: hydrostatic pressure force)

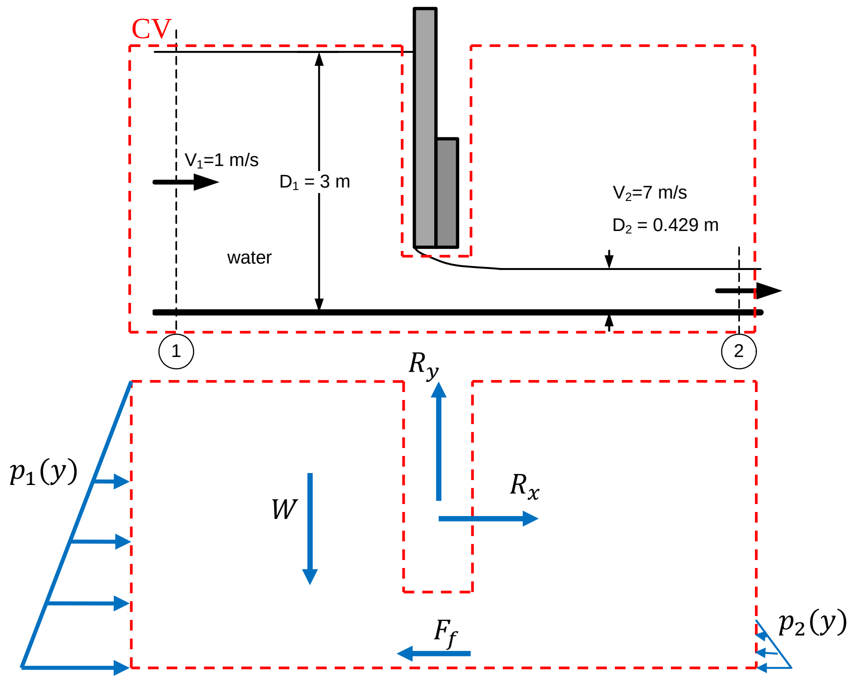
Water in an open channel is held in by a sluice gate. Compare the horizontal force of the water on the gate (a) when the gate is closed and (b) when it is open (assuming steady flow, as shown). Assume the flow at sections 1 and 2 is incompressible and uniform, and that (because the streamlines are straight there) the pressure distributions are hydrostatic.



Solution:

The forces acting on the control volume include:

- Force of gravity, W
- Friction force, F_f
- Components of R_x and R_y of reaction force from gate
- Hydrostatic pressure distribution on vertical surface
- Pressure distribution along the bottom surface (not shown)



Assumptions:

- Friction force F_f is negligible.
- No body force in x direction.
- Steady flow.
- Incompressible flow.
- Uniform flow at each section.
- Hydrostatic pressure distribution at sections 1 and 2.

Then,

$$F_{R1} + F_{R2} + R_x = (-\rho V_1^2 w D_1) + (\rho V_2^2 w D_2)$$

The surface forces acting on the CV are due to the pressure distributions and the unknown force, R_x. From our assumptions, we can integrate the gage pressure distribution on each side to compute the hydrostatic forces F_{R1} and F_{R2},

$$F_{R1} = \int_0^{D_1} p_1(y)\, dA = w\int_0^{D_1} \rho g y dy = \rho g w \frac{D_1^2}{2}$$

$$F_{R2} = \int_0^{2} p_2(y)\, dA = w\int_0^{D_2} \rho g y dy = \rho g w \frac{D_2^2}{2}$$

Hence,

$$R_x = \rho w (V_2^2 D_2 - V_1^2 D_1) - \frac{\rho g w}{2}(D_1^2 - D_2^2)$$

The horizontal force per unit width of the gate is

$$\frac{R_x}{w} = \rho (V_2^2 D_2 - V_1^2 D_1) - \frac{\rho g}{2}(D_1^2 - D_2^2)$$

Part (a):
When gate is closed, $D_2 = 0$ and $V_2 = 0$. From mass conservation $V_1 = 0$

$$\frac{R_x}{w} = -\frac{\rho g D_1^2}{2}$$

$$\frac{R_x}{w} = -999\left[\frac{kg}{m^3}\right] \times 9.81\left[\frac{m}{s^2}\right] \times \frac{(3[m])^2}{2} = -44.1\ kN$$

Part (b):
$V_1 = 1\ [m/s]$, and $V_2 = 7\ [m/s]$, hence,

$$\frac{R_x}{w} = 999\left[\frac{kg}{m^3}\right] \times \left[\left(7\ \left[\frac{m}{s}\right]\right)^2 \times (0.429[m]) - \left(1\ \left[\frac{m}{s}\right]\right)^2 \times (3[m])\right] - \frac{1}{2} \times 999\left[\frac{kg}{m^3}\right] \times 9.81\left[\frac{m}{s^2}\right] \times [(3[m])^2 - (0.429[m])^2] = -25.2\ kN$$

Note: As can be seen, the force on the open gate is significantly less than the force on the closed gate as the water accelerates out under the gate.