

## ENSC-283

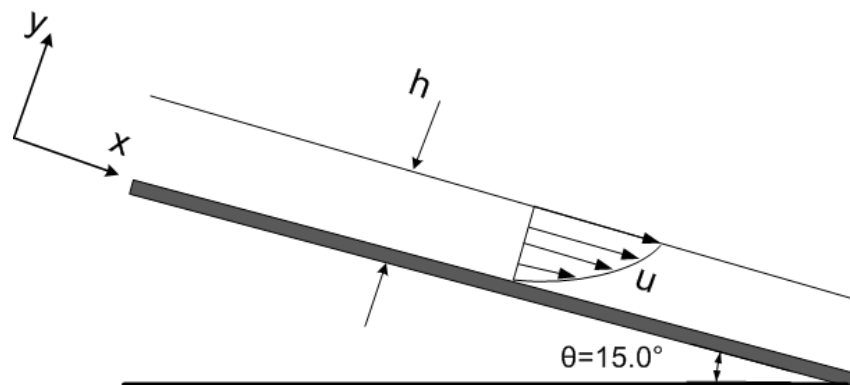
### Assignment #7

Assignment date: Monday Mar. 9, 2009

Due date: Monday Mar. 16, 2009

**Problem1:** (Fully developed laminar flow down an inclined plane surface)

A liquid flows down an inclined plane surface in a steady, fully developed laminar film of thickness  $h$ . Simplify the continuity and Navier-Stokes equations to model this flow field. Obtain expressions for the liquid velocity profile, the shear stress distribution, the volume flow rate, and the average velocity. Relate the liquid film thickness to the volume flow rate per unit depth of surface normal to the flow. Calculate the volume flow rate in a film of water  $h = 1 \text{ mm}$  thick, flowing on a surface  $b = 1 \text{ m}$  wide, inclined at  $\theta = 15^\circ$  to the horizontal.



**Problem2:** (analysis of laminar volumetric flow between concentric cylinders)

A viscous liquid fills the annular gap between vertical concentric cylinders. The inner cylinder is stationary, and the outer cylinder rotates at constant speed. The flow is laminar. Simplify the continuity, Navier-Stokes, and tangential shear stress equations to model this flow field. Obtain expressions for the liquid velocity profile and the shear stress distribution. Compare the shear stress at the surface of the inner cylinder with that computed from a planar approximation obtained by "unwrapping" the annulus into a plane and assuming linear velocity profile across the gap. Determine the ratio of cylinder radii for which the planar approximation predicts the correct shear stress at the surface of the inner cylinder within 1 percent.

