ENSC-283

Assignment #8

Assignment date: Monday Mar. 23, 2009

Due date: Monday Mar. 30, 2009

Problem1: (Noncircular conduit)

Air at a temperature of $50℃$ and standard pressure flows from a furnace through a $20 - \text{cm} - \text{diameter}$ pipe with an average velocity of $3\, m/s$. It then passes through a transition section and into a square duct whose side is of length a. The pipe and duct surfaces are smooth. Determine the duct size, a if the head loss per meter is to be the same for the pipe and the duct.

Solution:

We first have to calculate the flow rate:

$$Re = \frac{VD}{\nu} = \frac{3 \times 0.2}{15.69 \times 10^{-6}} = 37600$$

Hence, the flow is turbulent. With this Reynolds number and for a smooth tube and using the Moody chart results in $f = 0.022$, so that

We know that the friction loss per unit length of the pipe is,

$$\left(\frac{h_f}{L}\right)_{pipe} = \frac{fV^2}{2Dg} = \frac{0.22 \times 3^2}{2 \times 9.81 \times 0.2} = 0.02 = \left(\frac{h_f}{L}\right)_{square}$$

Hydraulic diameter for a square is

$$D_h = \frac{4A}{P} = \frac{4 \times a^2}{4a} = a$$

and

$$\left(\frac{h_f}{L}\right)_{square} = \frac{fV_s^2}{2D_h g} \qquad (1)$$

Average velocity in the square, V_s, can be obtained from the following relationship

$$V_s = \frac{Q}{A} = \frac{\frac{\pi}{4} \times 0.2^2 \times 3}{a^2} = \frac{0.094}{a^2}$$

Substituting in Eq. (1) yields

$$\left(\frac{h_f}{L}\right)_{square} = \frac{4.53 \times 10^{-4} f}{a^5} = 0.02$$

$$a = 0.468 f^{1/5} \qquad (2)$$

Similarly, the Reynolds number based on the hydraulic diameter is

$$Re_{D_h} = \frac{V_s D_h}{\nu} = \frac{\frac{0.094}{a^2} \times a}{15.69 \times 10^{-6}} = \frac{5994}{a} \qquad (3)$$

We have three unknowns, a , f and Re_{D_h}. Two find these unknowns three equations are needed. We already have Eqs. (2) and (3). For the other equation, Moody chart or Colebrook equation can be used. Using the Colebrook equation and letting $e = 0$ yields

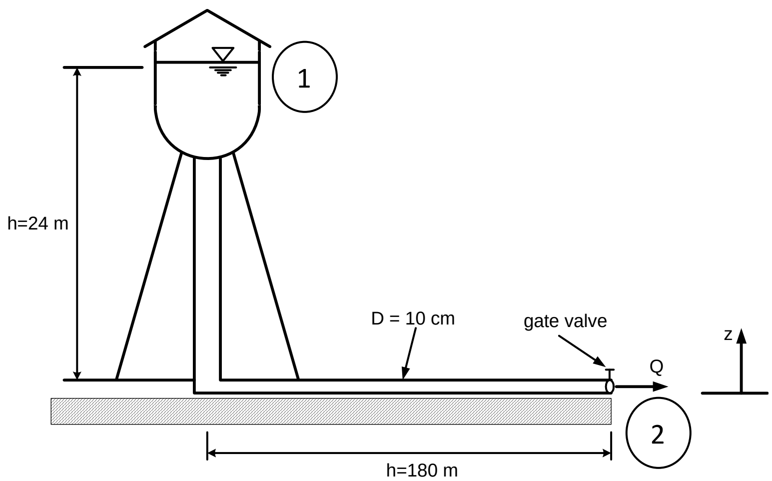
$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{Re_{D_h}\sqrt{f}}\right) \qquad (4)$$

Solving Eqs. (2) to (4) simultaneously,

$$f = 0.024;\ Re_{D_h} = 27000;\ a = 0.222$$

Problem 2: (Flow from a water tower: flow rate unknown)

A fire protection system is supplied from a water tower and standpipe $24\ m$ tall. The longest pipe in the system is $180\ m$ and is made of cast iron about 20 years old. The pipe contains one gate valve; other minor losses maybe neglected. The pipe diameter is $10\ cm$. Determine the maximum rate of flow (m^3/s) through this pipe.



Solution:

Writing energy equation between points 1 and 2 gives

$$\frac{p_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 + h_{l,t} \qquad (1)$$

where

$$h_{l,t} = h_l + h_{l,m};\quad h_l = f\frac{L}{D}\frac{V^2}{2};\ \ h_{l,m} = f\frac{L_e}{D}\frac{V^2}{2} \qquad (2)$$

Assuming that

$$p_1 = p_2 = p_{atm}$$

$$V_1 = 0;\ \alpha_2 \cong 1$$

Then Eq.(1) can be written as

$$g(z_1 - z_2) = \frac{V_2^2}{2} + \frac{f}{D}(L + L_e)\frac{V_2^2}{2} \qquad (3)$$

For fully open valve, $L_e/D = 8$. Substituting in Eq. (3) and after simplification gives

$$V_2 = \sqrt{\frac{2g(z_1 - z_2)}{f\left(\frac{L}{D} + 8\right) + 1}} \qquad (4)$$

To be conservative, assume the standpipe is the same diameter as the horizontal pipe. Then

$$\frac{L}{D} = \frac{180 + 24}{0.1} = 2040$$

To find f in Eq. (2), we first assume that the flow is fully turbulent. The roughness ratio, $e/D \approx 0.005$ ($\varepsilon = 0.00085$ for cast iron and doubled to allow for the fact that the pipe is old), Hence from the Colebrook equation

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.005}{3.7} + \frac{2.51}{Re_{D_h}\sqrt{f}}\right)$$

where

$$Re_{D_h} = \frac{V_2 D}{\nu} = \frac{V_2 \times 0.1}{10^{-6}} = 10^5 V_2$$

From Eq. (4),

$$V_2 = \sqrt{\frac{2 \times 9.81 \times 24}{f(2040 + 8) + 1}} = \frac{21.7}{\sqrt{2048f + 1}}$$

Hence the Reynolds number can be obtained from the following relationship

$$Re_{D_h} = \frac{21.7 \times 10^5}{\sqrt{2048f + 1}}$$

Substituting in the Colebrook equation and solving for f yields

$$f = 0.0307$$

and $V_2 = 2.71\ m/s$ and

$$Q = V_2 A = V_2 \frac{\pi D^2}{4} = 0.0213 \frac{m^3}{s}$$