ENSC-283

Assignment #9

Assignment date: Monday Mar. 30, 2009

Due date: Monday Apr. 6, 2009

Problem1: (Pressure drop in pipe flow)

The pressure drop, Δp, for steady, incompressible viscous flow through a straight horizontal pipe depends on the pipe length, l, the average velocity, V, the fluid viscosity, μ, the pipe diameter, D, the fluid density, ρ, and the average roughness, e height. Determine a set of dimensionless groups that can be used to correlate data.

Solution:

We know that

$$\Delta p = f(\rho, V, D, l, \mu, e)$$

This procedure should be followed for determining dimensionless Π parameters.

1- Determine number of dimensional parameters:

$$\Delta p, \rho, V, D, l, \mu, e \rightarrow n = 7$$

2- Choose primary dimensions, M, L and t.

Δp	ρ	V	D	l	μ	e
$\frac{M}{Lt^2}$	$\frac{M}{L^3}$	$\frac{L}{t}$	L	L	$\frac{M}{Lt}$	L

3- Select repeating parameters: $\rho, V, D \rightarrow$ number of primary dimensions: $r = 3$, and number of repeating parameters: $m = 3$. Hence, number of dimensionless groups will be $n - m = 4$.

4- Setting up dimensional equation yields

$$\Pi_1 = \rho^a V^b D^c \Delta p \text{ and}$$

$$\left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \left(\frac{M}{Lt^2}\right) = M^0 L^0 t^0$$

$$M: \quad 0 = a + 1$$

$$L: \quad 0 = -3a + b + c - 1$$

$$t: \quad 0 = -b - 2$$

$$\rightarrow a = -1, b = -2, c = 0$$

Therefore,

$$\Pi_1 = \frac{\Delta p}{\rho V^2}$$

Similarly

$$\Pi_2 = \rho^d V^e D^f \mu \text{ and}$$

$$\left(\frac{M}{L^3}\right)^d \left(\frac{L}{t}\right)^e L^f \left(\frac{M}{Lt}\right) = M^0 L^0 t^0$$

$$M: \quad 0 = d + 1$$

$$L: \quad 0 = -3d + e + f - 1$$

$$t:\ 0 = -e - 1$$

$$\rightarrow d = -1, e = -1, f = -1$$

Therefore,

$$\Pi_2 = \frac{\mu}{\rho V D}$$

$$\Pi_3 = \rho^g V^h D^i l \text{ and}$$

$$\left(\frac{M}{L^3}\right)^g \left(\frac{L}{t}\right)^h L^i(L) = M^0 L^0 t^0$$

$$M:\quad 0 = g$$

$$L:\ \ 0 = -3g + h + i + 1$$

$$t:\ 0 = -h$$

$$\rightarrow g = 0, h = 0, i = -1$$

Therefore,

$$\Pi_3 = \frac{l}{D}$$

$$\Pi_4 = \rho^j V^k D^l e \text{ and}$$

$$\left(\frac{M}{L^3}\right)^j \left(\frac{L}{t}\right)^k L^l(L) = M^0 L^0 t^0$$

$$M: \quad 0 = j$$

$$L: \; 0 = -3j + k + l + 1$$

$$t: \; 0 = -k$$

$$\rightarrow j = 0, k = 0, l = -1$$

Therefore,

$$\Pi_4 = \frac{e}{D}$$

The final relationship is

$$\frac{\Delta p}{\rho V^2} = \mathrm{f}\left(\frac{\mu}{\rho V D}, \frac{l}{D}, \frac{e}{D}\right)$$

Problem 2: (Similarity)

The drag of a sonar transducer is to be predicted, based on wind tunnel test data. The prototype, 0.3 m diameter sphere, is to be towed at 3 m/s in sea water at 5℃. The model is $10\ cm$ in diameter. Determine the required test speed in air. If the drag of the model at these test condition is $0.25\ N$, estimate the drag of the prototype.

Solution:

Since the prototype operates in water and the model test is to be performed in air, useful results can be expected only if cavitation effects are absent in the prototype flow and compressibility effects are absent from the model test. Under these conditions,

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\rho V D}{\mu}\right)$$

And the test should be run at

$$Re_m = Re_p$$

to ensure dynamic similarity. For seawater at 5℃, $\rho \cong 1000\ kg/m^3$ and $\mu \cong 0.0015\ Pa.s$. At prototype conditions,

$$V_p = 3\frac{m}{s}$$

$$Re_p = \frac{\rho_p V_p D_p}{\mu_p} = \frac{1000 \times 3 \times 0.3}{0.0015} = 600000$$

Therefore,

$$Re_m = \frac{\rho_m V_m D_m}{\mu_m} = Re_p \rightarrow V_m = \frac{\mu_m Re_p}{\rho_m D_m} = \frac{1.75 \times 10^{-5} \times 6 \times 10^5}{1.25 \times 0.1} = 84\frac{m}{s}$$

This speed is low enough to neglect compressibility effects, $Ma < 0.3$.

As these test conditions, the model and prototype flows are dynamically similar. Hence

$$\left(\frac{F}{\rho V^2 D^2}\right)_m = \left(\frac{F}{\rho V^2 D^2}\right)_p \rightarrow F_p = \frac{\rho_p}{\rho_m}\frac{V_p^2}{V_m^2}\frac{D_p^2}{D_m^2} = 2.3\ N$$