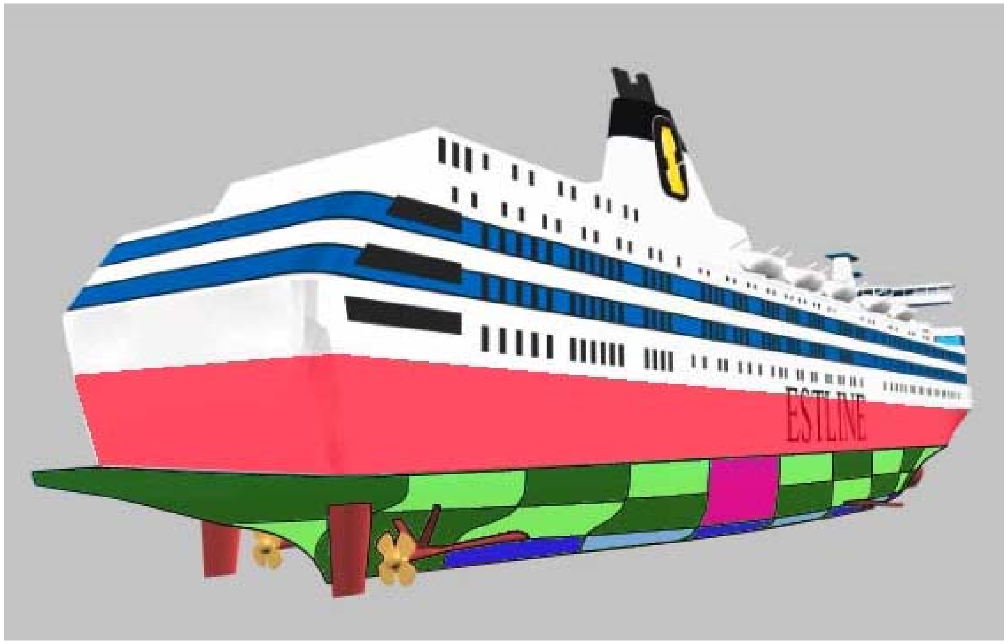
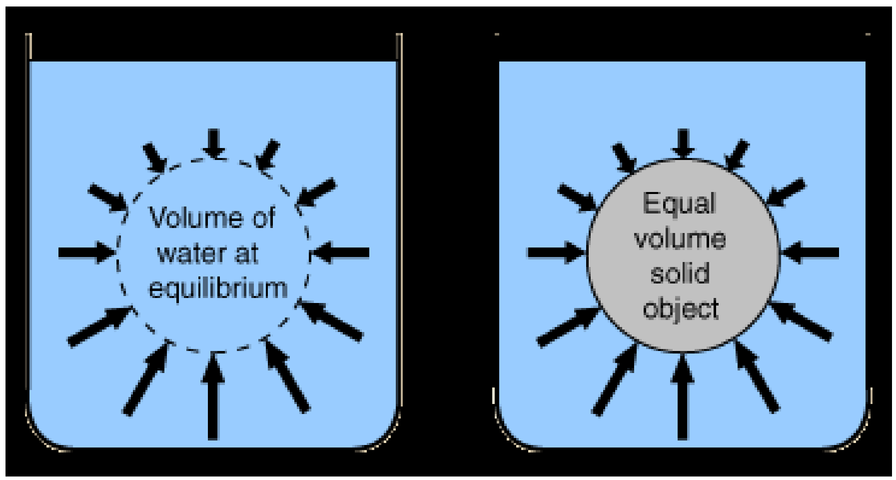
Buoyancy

- From Buoyancy principle, we can see whether an object floats or sinks. It is based on not only its weight, but also the amount of water it displaces. That is why a very heavy ocean liner can float. It displaces a large amount of water.

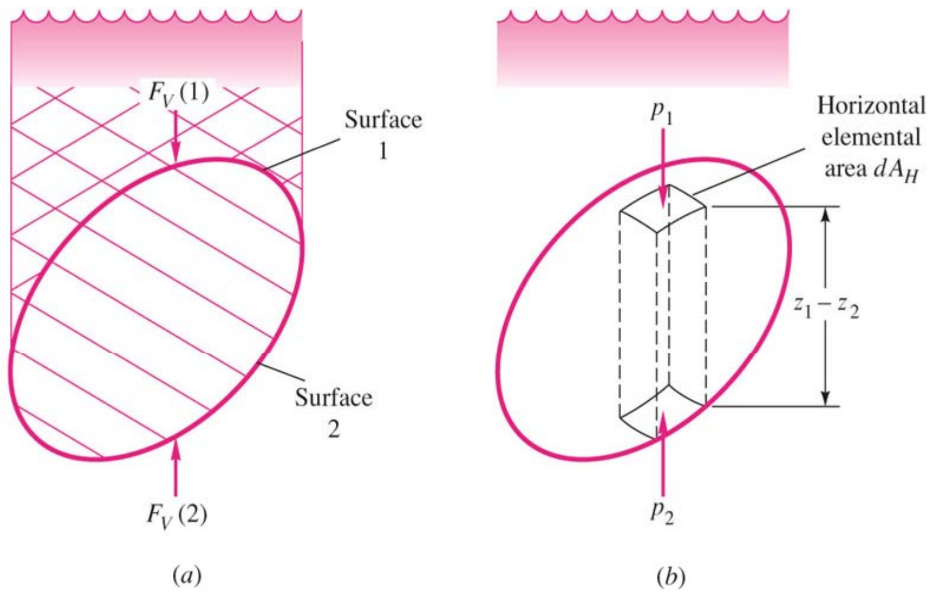


Archimedes 1st law

- A body <u>immersed</u> in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces



Buoyancy force

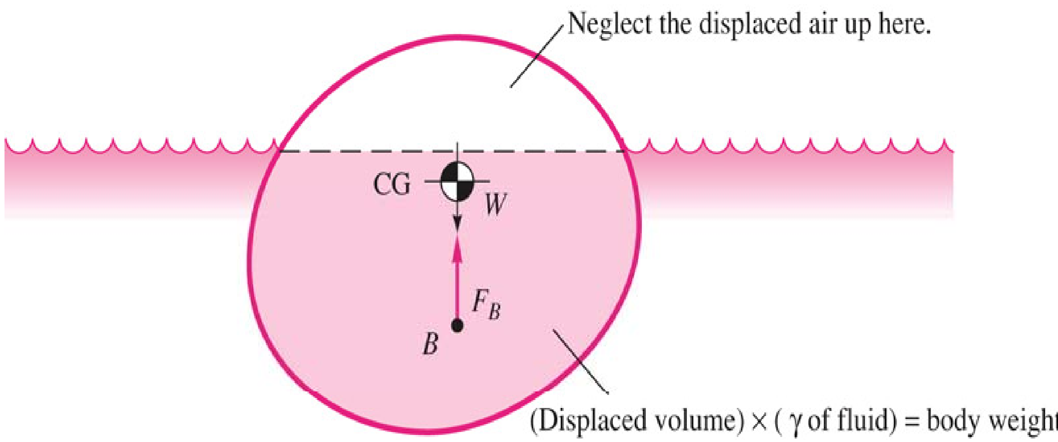


$$F_B = \int_{body} (p_2 - p_1)\, dA_H = -\gamma \int (z_2 - z_1)\, dA_H = \gamma(body\ volume)$$

- The line of action of the buoyant force passes through the center of volume of the displaced body; i.e., the center of mass is computed as if it had uniform density. The point which F_B acts is called the *center of buoyancy*.

Archimedes 2nd law

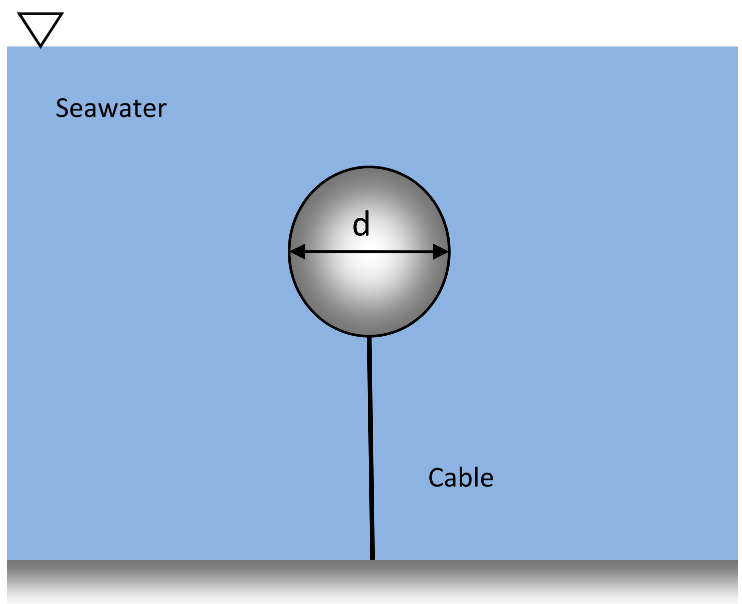
- A floating body displaces its own weight in the fluid in which it floats. In the case of a floating body, only a portion of the body is submerged.

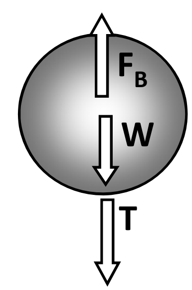


$$F_B = \gamma(displaced\ volume) = \text{weight of the floating body}$$

Example

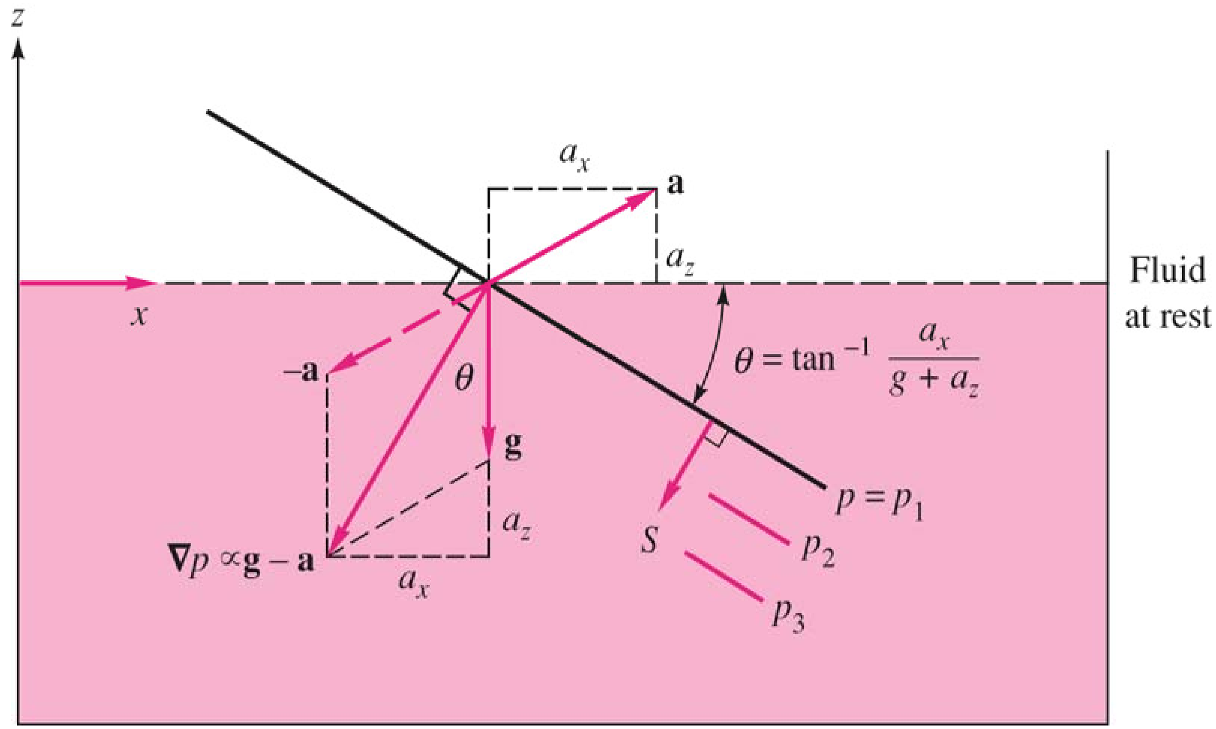
- A spherical body has a diameter of 1.5 *m*, weighs 8.5 *kN*, and is anchored to the sea floor with a cable as is shown in the figure. Calculate the tension of the cable when the body is completely immersed, assume $\gamma_{sea\text{-}water}$ =10.1 kN/m^3.





Pressure in rigid-body motion

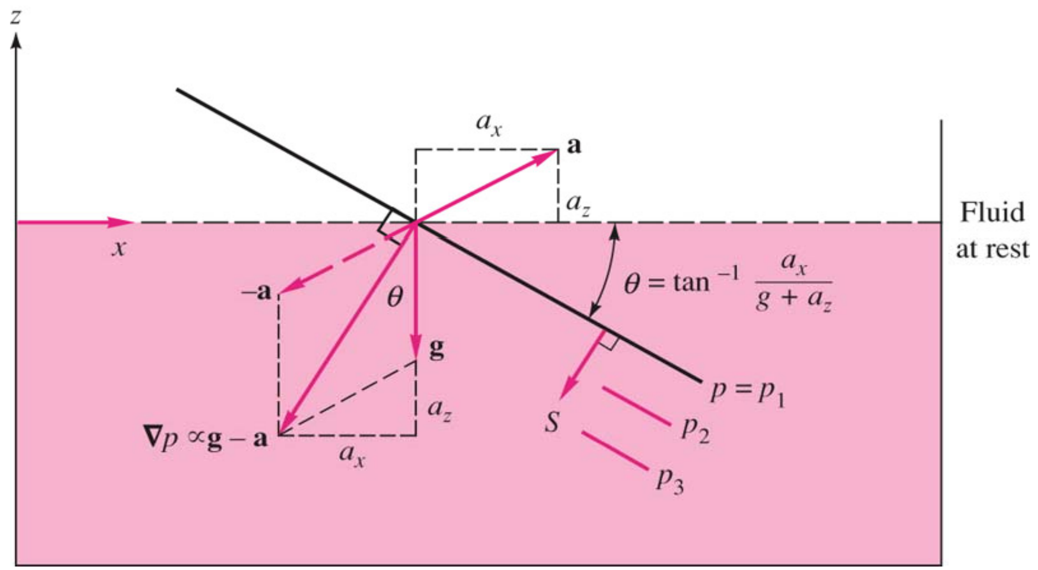
- Fluids move in rigid-body motion only when restrained by confining walls. In rigid-body motion, all particles are in combined translation and rotation, and there is no relative motion between particles.



$$\nabla p = \rho(\vec{g} - \vec{a})$$

Rigid-body motion cont'd

- The pressure gradient acts in the direction of g – a and lines of constant pressure are perpendicular to this direction and thus tilted at angle θ



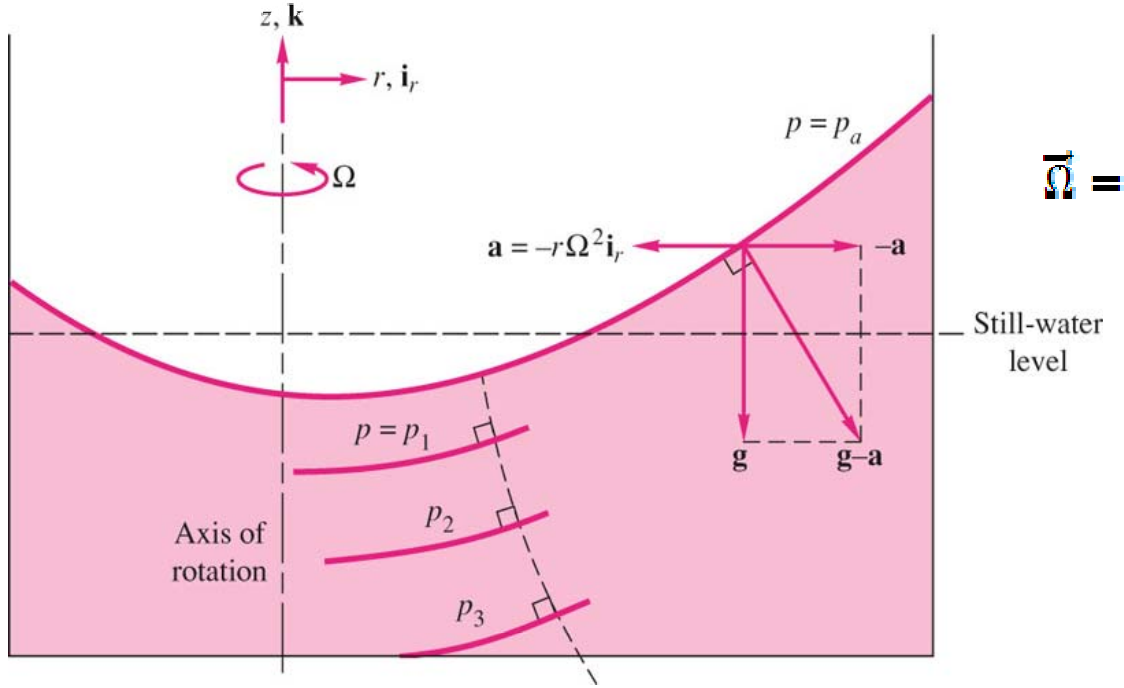
$$\theta = \tan^{-1}\frac{a_x}{g + a_z}$$

- The rate of increase of pressure in the direction g – a is greater than in ordinary hydrostatics

$$\frac{dp}{ds} = \rho G \ \ where \ \ G = [a_x^2 + (g + a_z)^2]^{1/2}$$

Rigid-body rotation

- Consider a fluid rotating about the z-axis without any translation at a constant angular velocity Ω for a long time.



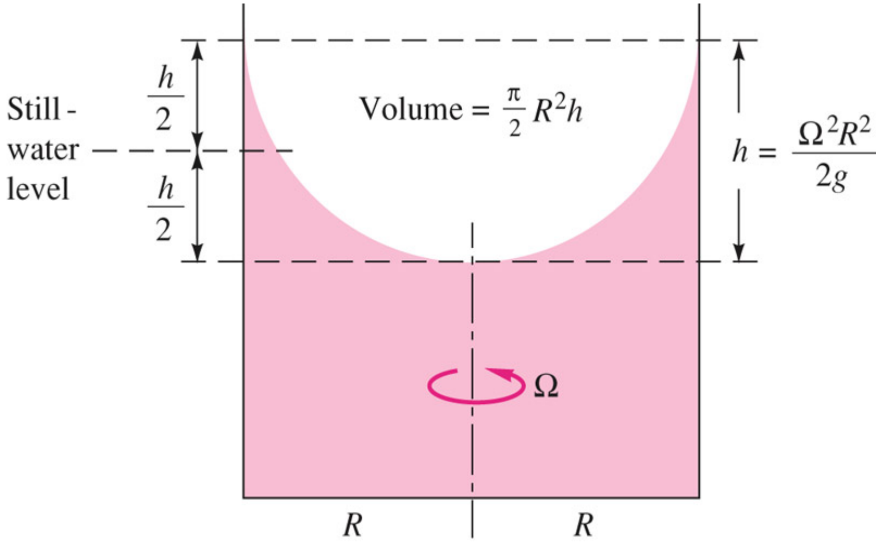
$$\vec{\Omega} = \Omega k \qquad \vec{r} = r i_r$$

The acceleration id given by: $\Omega \times (\Omega \times \vec{r}) = -r\Omega^2 i_r$

The forced balance becomes: $\nabla p = \frac{\partial p}{\partial r} i_r + \frac{\partial p}{\partial z} k = \rho(g - a) = \rho(-gk + r\Omega^2 i_r)$

Rigid-body motion cont'd

The pressure field can be found by equating like components



$$\frac{\partial p}{\partial r} = \rho r \Omega^2 \qquad \frac{\partial p}{\partial z} = -\gamma$$

- After integration with respect to *r* and *z* with $p=p_0$ at $(r,z) = (0,0)$:

$$p = p_0 - \gamma z + \frac{1}{2}\rho r^2 \Omega^2$$

- The pressure is linear in z and parabolic in r. The constant pressure surfaces can be calculated using

$$z = \frac{p_0 - p_1}{\gamma} + \frac{r^2 \Omega^2}{2g} = a + br^2$$

Pressure measurement

- Pressure is the force per unit area and can be imagined as the effects related to fluid molecular bombardment of a surface.
- There are many devices for both a static fluid and moving fluid pressure measurements. Manometer, barometer, Bourdon gage, McLeod gage, Knudsen gage are only a few examples.