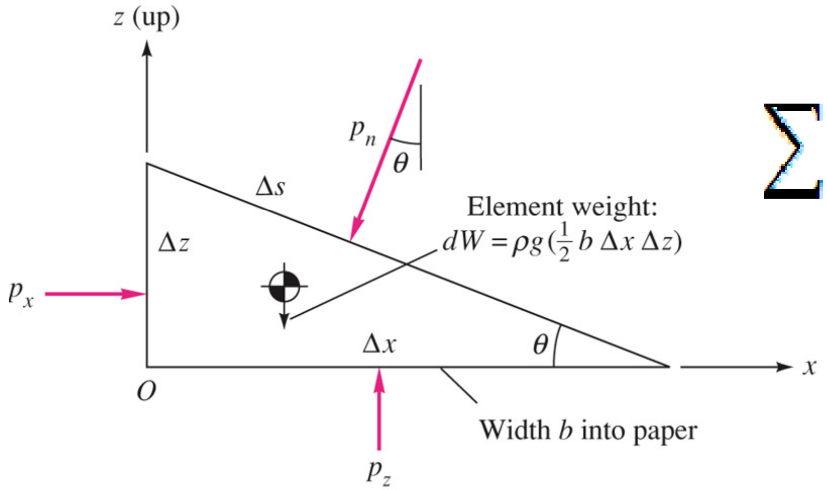
Hydrostatic

- Pressure distribution in a static fluid and its effects on solid surfaces and on floating and submerged bodies.

Fluid at rest

- hydrostatic condition: when a fluid velocity is zero, the pressure variation is due only to the weight of the fluid.



$$\sum F_z = 0 = p_z\, b\, \Delta x - p_n b\, \Delta s\, \cos\theta - \frac{1}{2}\rho g\, b\, \Delta x\, \Delta z$$

$$\sum F_x = 0 = p_x\, b\, \Delta z - p_n b\, \Delta s\, \sin\theta$$

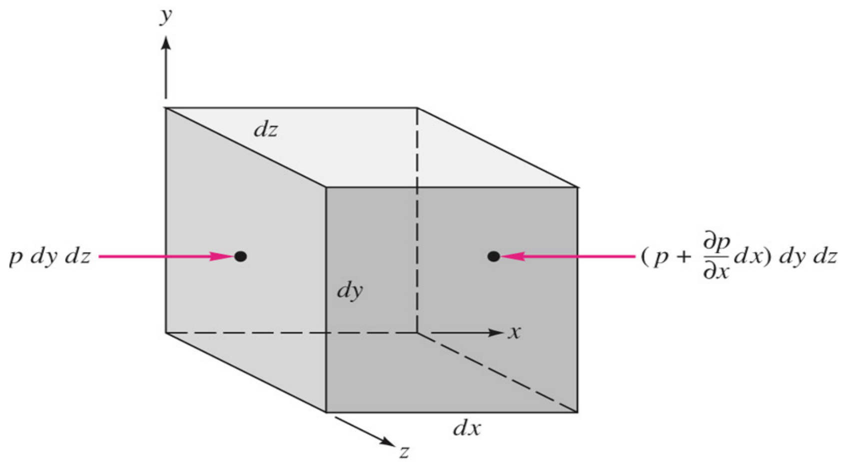
$$p_x = p_n \qquad p_z = p_n + \frac{1}{2}\rho g\, \Delta z$$

- There is no pressure change in the horizontal direction.
- There is a pressure change in the vertical direction proportional to the density, gravity, and depth change.

- In the limit when the wedge shrinks to a point, $p_x = p_z = p_n = p$

Pressure forces (pressure gradient)

- Assume the pressure vary arbitrarily in a fluid, $p=p(x,y,z,t)$.



$$dF_x = p\,dy\,dz - \left(p + \frac{\partial p}{\partial x} dx\right) dy\,dz = -\frac{\partial p}{\partial x} dx\, dy\, dz$$

$$dF_{press} = -\left(\frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\right) dx\, dy\, dz$$

$$\boldsymbol{f}_{press} = -\nabla p$$

- The pressure gradient is a <u>surface force</u> that acts on the sides of the element.
- Note that the pressure gradient (not pressure) causes a net force that must be balanced by gravity or acceleration.

Equilibrium

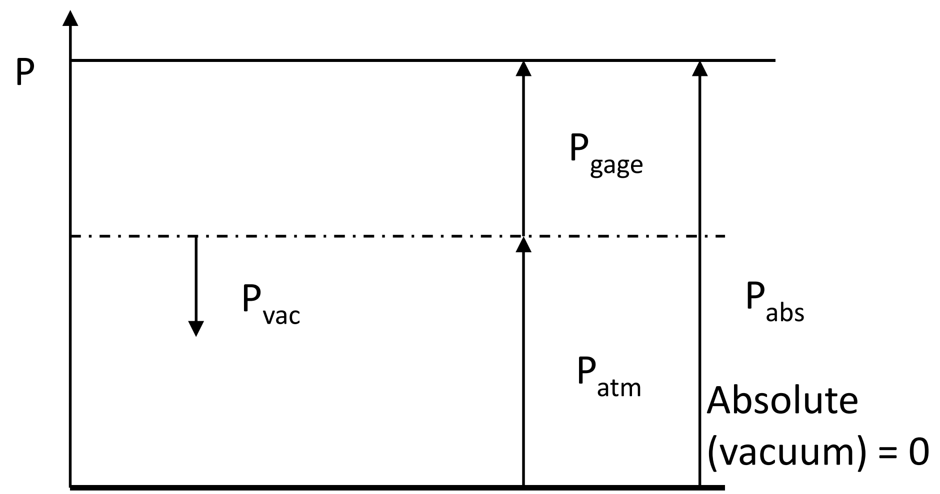
- The pressure gradient must be balanced by gravity force, or weight of the element, for a fluid at rest.

$$dF_{gravity} = \rho g dx\, dy\, dz \qquad f_{gravity} = \rho g$$

- The gravity force is a body force, acting on the entire mass of the element. Magnetic force is another example of body force.

Gage pressure and vacuum

- The actual pressure at a given position is called the absolute pressure, and it is measured relative to absolute vacuum.



$p > p_a$ *Gage pressure* $p_{gage} = p - p_a$

$p < p_a$ *Vacuum pressure* $p_{vacuum} = p_a - p$

Hydrostatic pressure distribution

- For a fluid at rest, pressure gradient must be balanced by the gravity force

$$\nabla p = \rho \mathbf{g}$$

- Recall: ∇p is perpendicular everywhere to surface of constant pressure *p*.
- In our customary coordinate z is "upward" and the gravity vector is:

$$\mathbf{g} = -gk$$

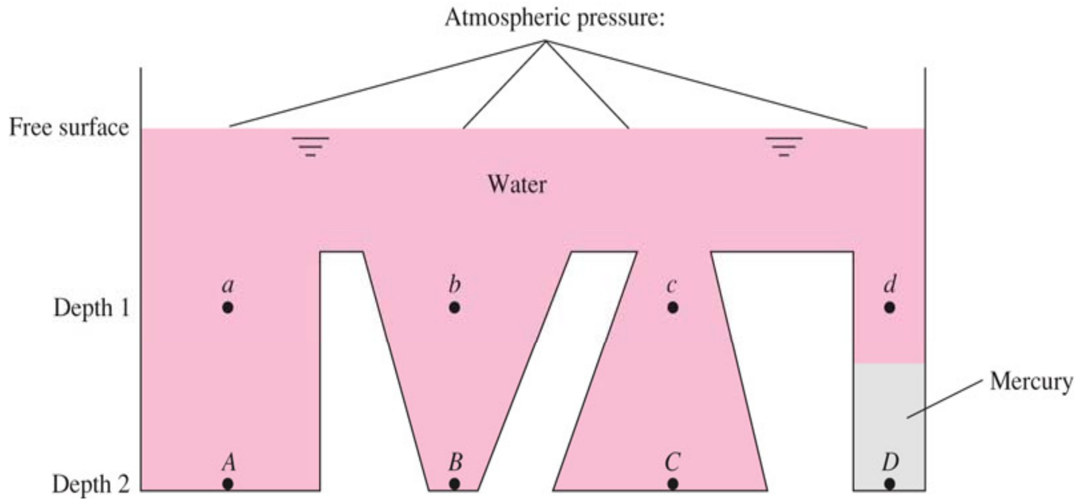
where g = 9.807 m/s^2. The pressure gradient vector becomes:

$$\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\rho g = -\gamma$$

$$\frac{dp}{dz} = -\gamma \qquad p_2 - p_1 = -\int_1^2 \gamma dz$$

Hydrostatic pressure distribution

- Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container.
- The pressure is the same at all points on a given horizontal plane in a fluid.



$$p_a = p_b = p_c$$

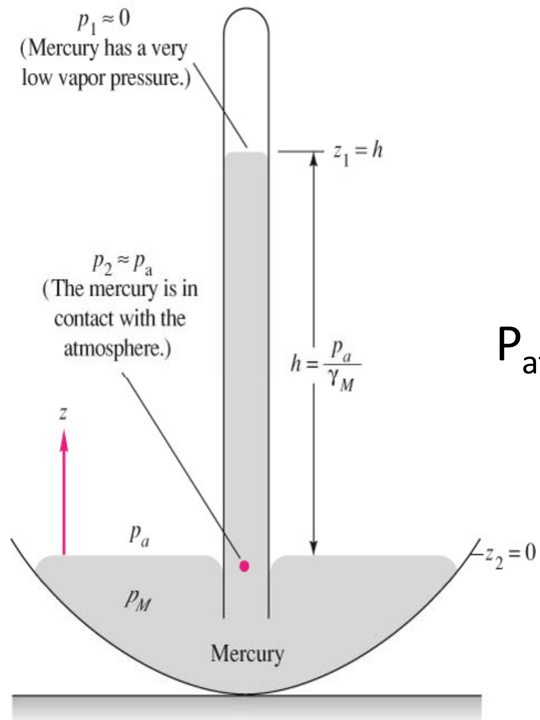
$$p_A = p_B = p_C \neq p_D$$

- For liquids, which are incompressible, we have:

$$p_2 - p_1 = -\gamma(z_2 - z_1) \quad or \quad z_1 - z_2 = \frac{p_2}{\gamma} - \frac{p_1}{\gamma}$$

- The quantity, p/γ is a length called the <u>pressure head</u> of the fluid.

The mercury barometer



(a)

P_{atm} = 761 mmHg

- Mercury has an extremely small vapor pressure at room temperature (almost vacuum), thus $p_1 = 0$. One can write:

$$p_a - 0 = -\gamma_{mercury}(0 - h) \quad or \quad h = \frac{p_a}{\gamma_{mercury}}$$

Hydrostatic pressure in gases

- Gases are compressible, using the ideal gas equation of state, $p = \rho R T$:

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT} g$$

- For small variations in elevation, "isothermal atmosphere" can be assumed:

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

- In general (for higher altitudes) the atmospheric temperature drops off linearly with *z*

$$T \approx T_0 - Bz$$

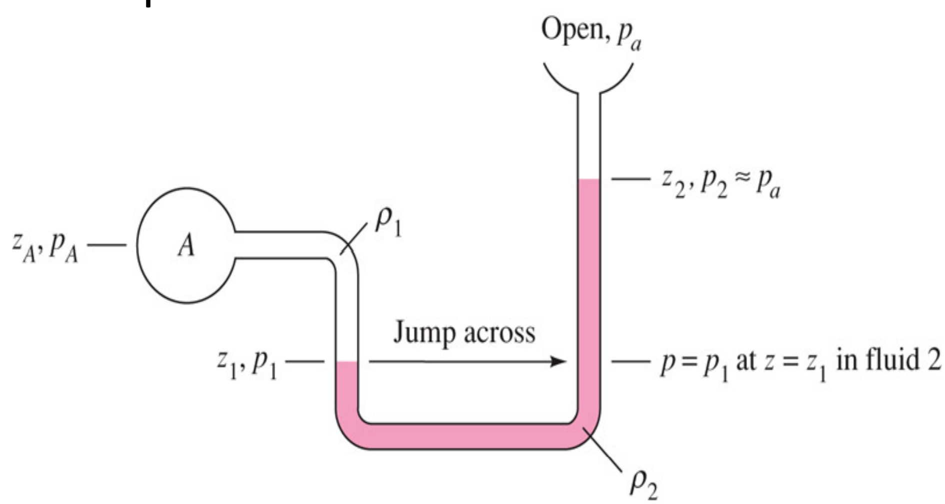
where T_0 is the sea-level temperature (in Kelvin) and B=0.00650 K/m.

$$p = p_a \left(1 - \frac{Bz}{T_0}\right)^{g/RB} \quad \text{for air } \frac{g}{RB} = 5.26$$

- Note that the P_{atm} is nearly zero (vacuum condition) at z = 30 km.

Manometry

- A static column of one or multiple fluids can be used to measure pressure difference between 2 points. Such a device is called manometer.



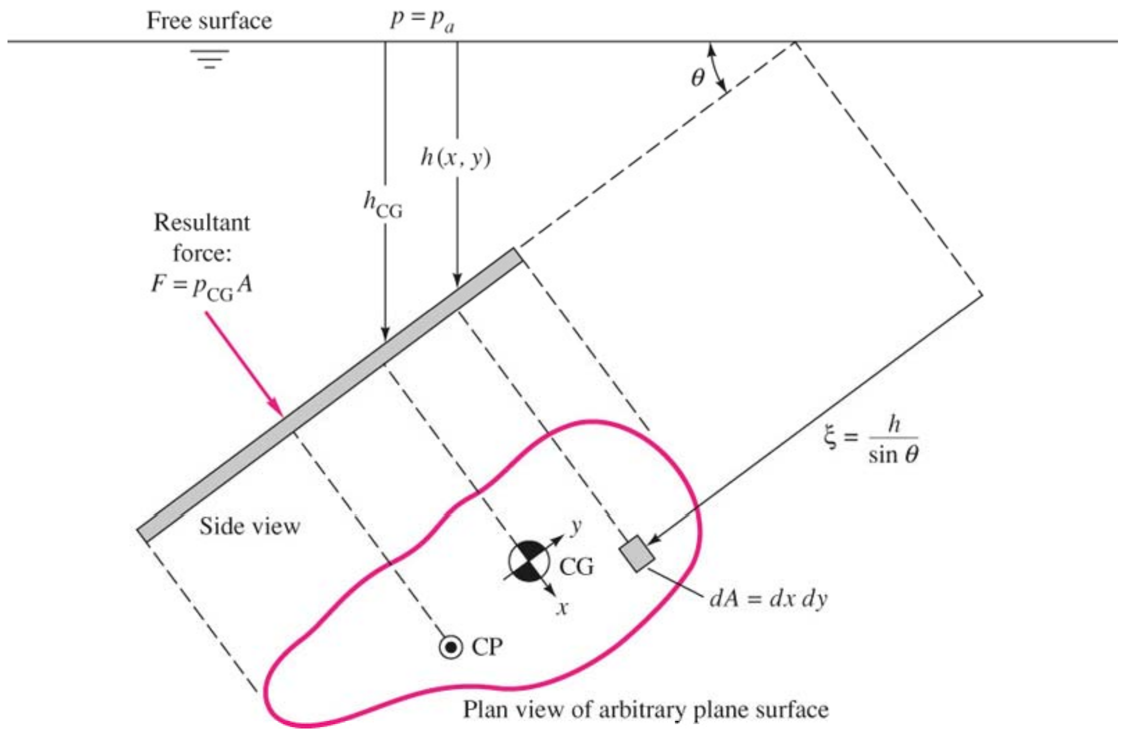
$$p_A + \gamma_1 |z_A - z_1| - \gamma_2 |z_1 - z_2| = p_2 = p_{atm}$$

- <u>Adding/ subtracting</u> $\gamma\Delta z$ as moving down/up in a fluid column.

- <u>Jumping across U-tubes</u>: any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure.

Hydrostatic forces on surfaces

- Consider a plane panel of arbitrary shape completely submerged in a liquid.



$$p = p_a + \gamma h$$

- The total hydrostatic force on one side of the plane is given by:

$$F = \int p dA = \int (p_a + \gamma h)\, dA = p_a A + \gamma \int h dA$$

Hydrostatic forces on surfaces

- After integration and simplifications, we find:

$$F = p_a A + \gamma h_{CG} A = (p_a + \gamma h_{CG})A = p_{CG} A$$

- The force on one side of any plane submerged surface in a uniform fluid equals the pressure at the plate centroid times the plate area, independent of the shape of the plate or angle θ.

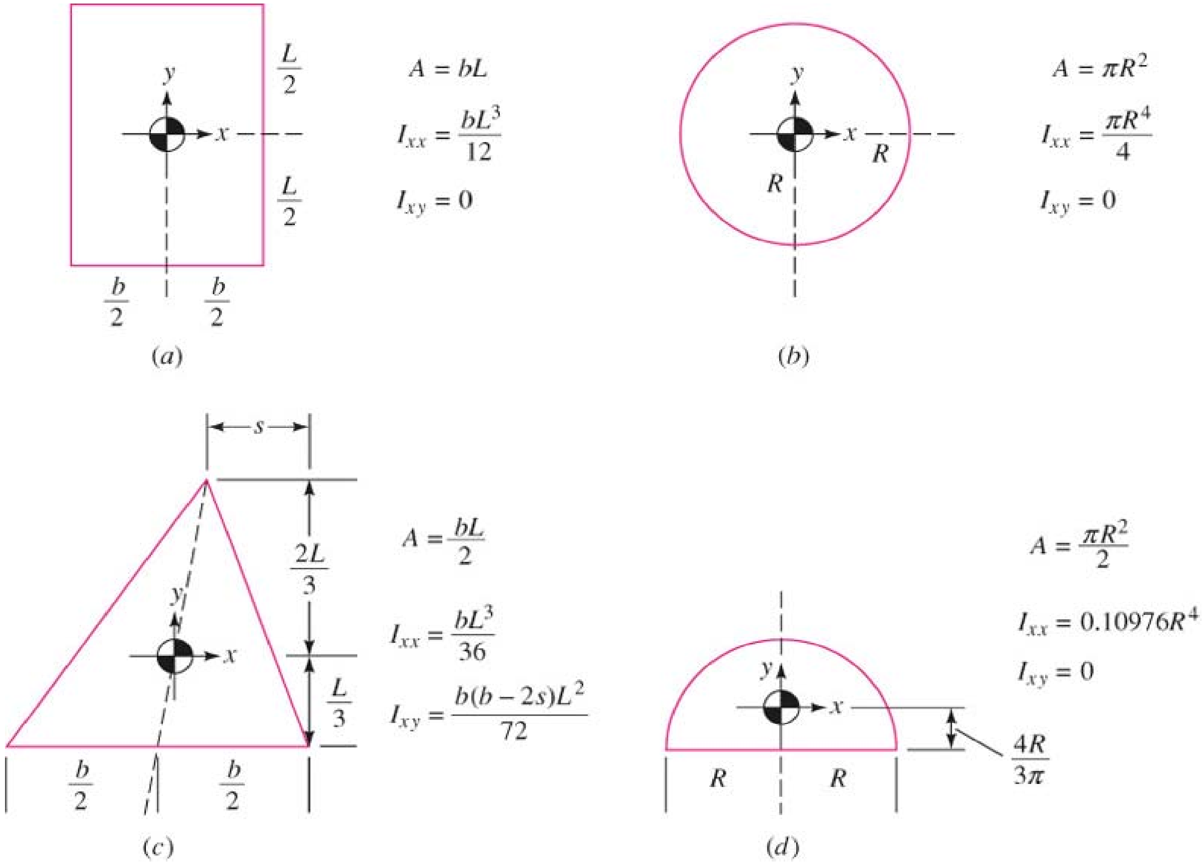
- The resultant force acts not through the centroid but below it toward the high pressure side. Its line of action passes through the centre of pressure CP of the plate (x_{CP}, y_{CP}).

$$F y_{CP} = \int y p dA = \int y(p_a + \gamma \xi sin\theta)\, dA = \gamma sin\theta \int y \xi dA$$

$$y_{CP} = -\gamma sin\theta \frac{I_{xx}}{p_{CG} A} \qquad x_{CP} = -\gamma sin\theta \frac{I_{xy}}{p_{CG} A}$$

Hydrostatic forces on surfaces

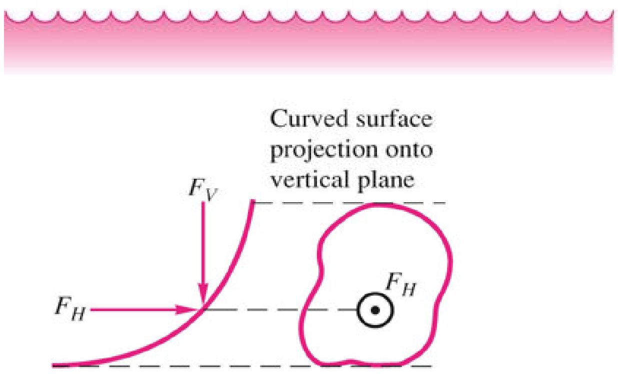
- Centroidal moments of inertia for various cross-sections.

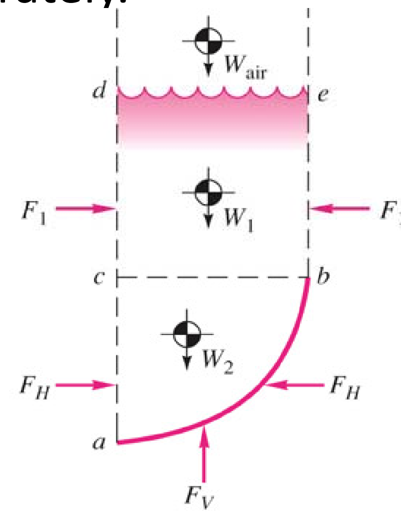


- Note: for symmetrical plates, $I_{xy} = 0$ and thus $x_{CP} = 0$. As a result, the center of pressure lies directly below the centroid on the y axis.

Hydrostatic forces: curved surfaces

- The easiest way to calculate the pressure forces on a curved surface is to compute the horizontal and vertical forces separately.





- The horizontal force equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.

- The vertical component equals to the weight of the entire column of fluid, both liquid and atmospheric above the curved surface.

$$F_V = W_2 + W_1 + W_{air}$$