Angular momentum equation

- For angular momentum equation, $B = H_O$ the angular momentum vector about point O which moments are desired.

$$H_O = \int_{System} (\vec{r} \times \vec{V})\, dm$$

- Where β is $\beta = \frac{dH_O}{dm} = \vec{r} \times \vec{V}$

- The Reynolds transport equation can be written as follows:

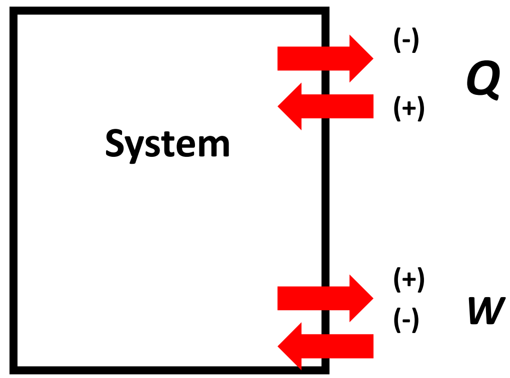
$$\frac{dH_O}{dt} = \sum M_O = \sum (\vec{r} \times \vec{F})_O = \frac{d}{dt}\left[\int_{CV} (\vec{r} \times \vec{V}) \rho dV\right] + \int_{CS} (\vec{r} \times \vec{V}) \rho (\vec{V}.\vec{n})\, dA$$

- For one-dimensional inlets and outlets, the flux terms become:

$$\int_{CS} (\vec{r} \times \vec{V}) \rho (\vec{V}.\vec{n})\, dA = \sum (\vec{r} \times \vec{V})_{out} \dot{m}_{out} - \sum (\vec{r} \times \vec{V})_{in} \dot{m}_{in}$$

Conservation of energy

- The conservation of energy principle or the first law of thermodynamics: during an interaction, energy can change from one form to another but the total amount of energy remains constant.



- A system can exchange energy with its surroundings through heat and work transfer. In other words, work and heat are the forms that energy can be transferred across the system boundary.

- <u>Sign convention</u>: work done by a system is positive, and the work done on a system is negative. Heat transfer to the system is positive and from a system will be negative.

First law of thermodynamics

- If δQ heat is added to a system and δW work is done by the system, the system energy dE must change based on the first law of thermodynamics:

$$\delta Q - \delta W = dE$$

- In a rate form:

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

- Applying the Reynolds transport theorem to the first law of thermodynamics, the dummy variable B=E (energy) and β=dE⁄(dm)= e. The Reynolds transport equation for a fixed control volume becomes:

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\left(\int_{CV} e\rho dV\right) + \int_{CS} e\rho(V.n)dA$$

- <u>Energy per unit mass</u>: can include: internal, potential, and kinetic:

$$e = u + \frac{1}{2}V^2 + gz$$

Work rate

- Work rate can be divided into three parts:

$$W = W_{shaft} + W_{press} + W_{viscous\ stresses} = W_s + W_p + W_v$$

- Shaft work: is the work done by a machine (e.g. pump impeller, fan blade, piston, etc) and it involves a shaft that crosses the control surface.

- Pressure work: is done by pressure forces and occurs at the surface only; all work on internal portions of the material in the control volume is by equal and opposite forces and is self-canceling:

$$dW_p = -(pdA)V_{n,in} = -p(-V.n)dA$$

- To find the total pressure work:

$$W_p = \int_{CS} p(V.n)dA$$

Work rate cont'd.

- <u>Shear work</u>: is due to viscous stresses occurs at the control surface and consists of the product of each viscous stress (one normal and two tangential) and the respective velocity component:

$$dW_v = -\tau . V dA \quad or \quad W_v = -\int_{CS} \tau . V dA$$

- Shear work is rarely important and may vanish or be negligible according to the particular type of surface.
- Note 1: that for all parts of control surface that are solid containing walls, V=0 (no-slip condition) thus W_v=0 .
- Note 2: for surface of a machine, the viscous work is contributed to the machine and we absorb it in the shaft work W_s.
- Note 3: the viscous work at inlet or outlets can be neglected due to negligible amount of $\tau_{nn} V_n dA$.

$$\dot{W} = \dot{W}_s + \int_{CS} p(V.n)dA - \int_{CS} (\tau . V)_{stream\ surface} dA$$

Energy equation

- The control volume energy equation becomes:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{d}{dt}\left(\int_{CV} e\rho dV \right) + \int_{CS} \left(e + \frac{p}{\rho} \right) \rho (V.n) dA$$

- From definition of enthalpy *h=u+p/ρ*, the final general form of the energy equation for a fixed control volume becomes:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{d}{dt}\left(\int_{CV} \left(u + \frac{1}{2}V^2 + gz \right) \rho dV \right) + \int_{CS} \left(h + \frac{1}{2}V^2 + gz \right) \rho (V.n) dA$$

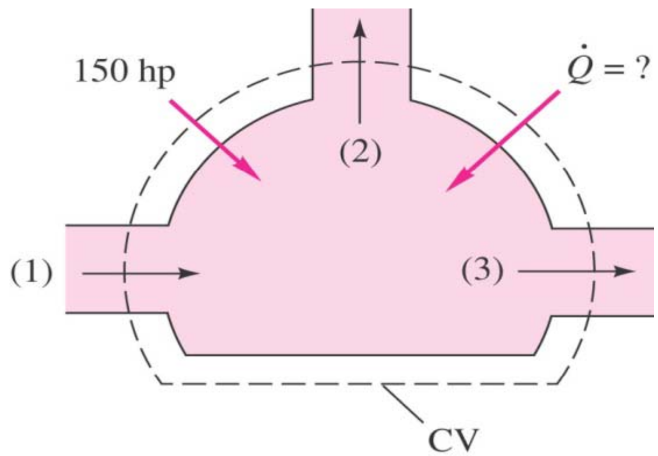
- For one-dimensional inlets and outlets, the flux terms become:

$$\int_{CS} \left(h + \frac{1}{2}V^2 + gz \right) \rho (V.n) dA = \sum \left(h + \frac{1}{2}V^2 + gz \right)_{out} \dot{m}_{out} - \sum \left(h + \frac{1}{2}V^2 + gz \right)_{in} \dot{m}_{in}$$

Example

- A steady flow machine, shown in figure below, takes in air at section 1 and discharges it at section 2 and 3. the properties at each section are as follows:

section	A, ft^2	Q, ft^3/s	T, F	p, lbf/in^2 abs	z, ft
1	0.4	100	70	20	1.0
2	1.0	40	100	30	4.0
3	0.25	50	200	?	1.5



- Work is provided to the machine at the rate of 150 hp. Find the pressure p3 in lbf/in^2 absolute and heat transfer in Btu/s. Assume air is a perfect gas with R=1716 and c_p = 6003 ft. lbf/(slug.R).

Steady flow energy equation

- For steady flow with one inlet and one outlet, both assumed one-dimensional, reduces to a celebrated relation used in many engineering analyses:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \left(h + \frac{1}{2}V^2 + gz\right)_{out} \dot{m}_{out} - \left(h + \frac{1}{2}V^2 + gz\right)_{in} \dot{m}_{in}$$

- From conservation of mass $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$ we can re-arrange it:

$$\left(h + \frac{1}{2}V^2 + gz\right)_{in} = \left(h + \frac{1}{2}V^2 + gz\right)_{out} - q + w_s + w_v$$

- The following term is called stagnation enthalpy:

$$H_1 = (h + 1/2\, V^2 + gz)_1$$

- If we divide the steady energy equation by g, the dimension of each term becomes a length, called head in fluid mechanics and shown by symbol *h*

$$\frac{p_1}{\gamma} + \frac{u_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{u_2}{g} + \frac{V_2^2}{2g} + z_2 - h_q + h_s + h_v$$

SFU

Friction and shaft work

- The energy equation for steady, low speed, incompressible flows through a pipe or a duct with a pump or a turbine which has one inlet and one outlet becomes:

$$\left(\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1\right)=\left(\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\right)+h_{friction}-h_{pump}+h_{turbine}$$

- Kinetic energy correction factor: The inlet and outlet to control volume often do not have one-dimensional flow, i.e., flow varies across the cross-section. To use the average velocity in KE calculation, we should introduce:

$$\int_{port}\frac{V^2}{2}\rho(V.n)\,dA\equiv\alpha\frac{V_{av}^2}{2}\dot{m}\quad where\quad V_{av}=\frac{1}{A}\int u\,dA$$

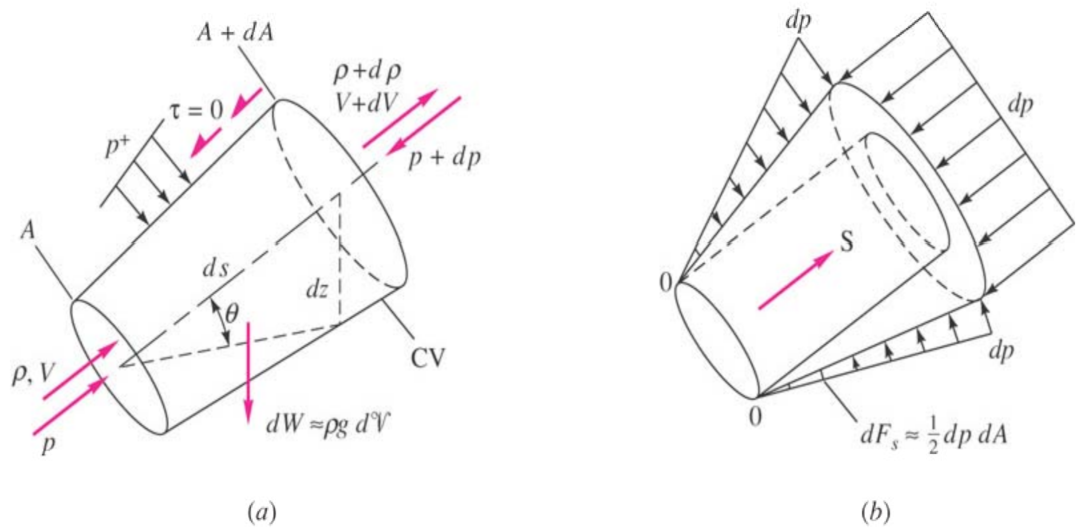
$$u=U_0\left(1-\frac{r^2}{R^2}\right)\quad with\ \alpha_{laminar}=2.0\qquad u=U_0\left(1-\frac{r}{R}\right)^m\quad with\ \alpha_{turbulent}=\frac{(2+m)^3(1+m)^3}{4(1+3m)(2+3m)}$$

- Introducing the kinetic correction factor, the steady energy equation for incompressible flows becomes:

$$\left(\frac{p_1}{\gamma}+\frac{\alpha V_1^2}{2g}+z_1\right)=\left(\frac{p_2}{\gamma}+\frac{\alpha V_2^2}{2g}+z_2\right)+h_{friction}-h_{pump}+h_{turbine}$$

Bernoulli equation

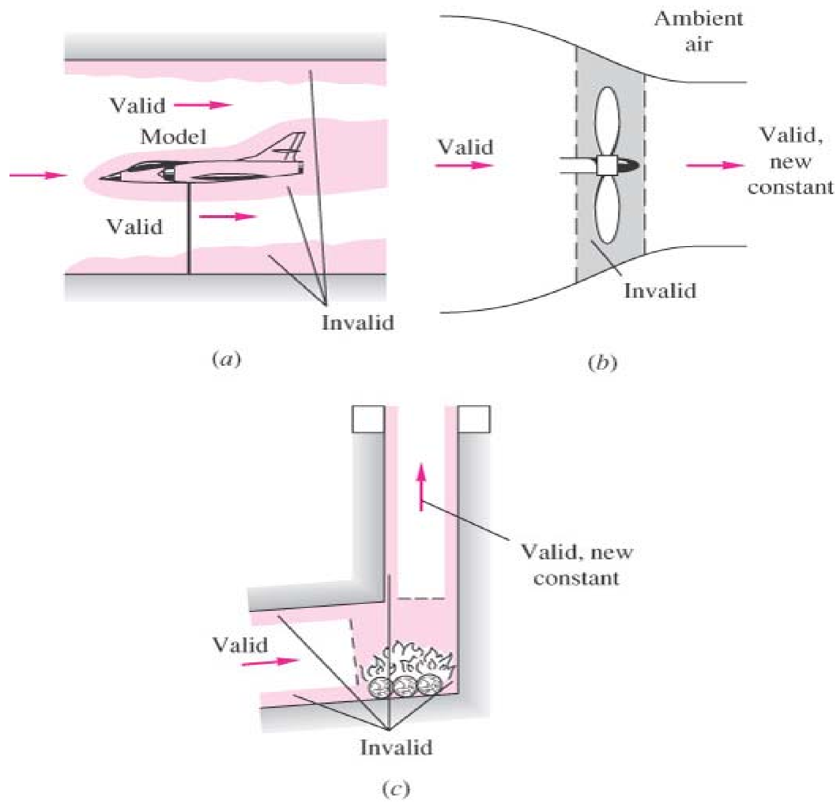
- If flow is assumed frictionless, the steady energy equation reduced to the Bernoulli equation.



$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const.$$

- The Bernoulli for <u>steady frictionless incompressible flow along a streamline.</u>

Application of Bernoulli equation



- It should be noted that different streamlines may have different flow conditions and different Bernoulli constants.

$$\text{Bernoulli constant} \equiv \frac{p}{\rho} + \frac{1}{2}V^2 + gz$$

Bernoulli vs. energy equation

- A comparison between the steady flow energy and Bernoulli equation reveals that the energy equation is much more general than Bernoulli and allows for i) friction, ii) heat transfer, iii)shaft work, and iv) various work.

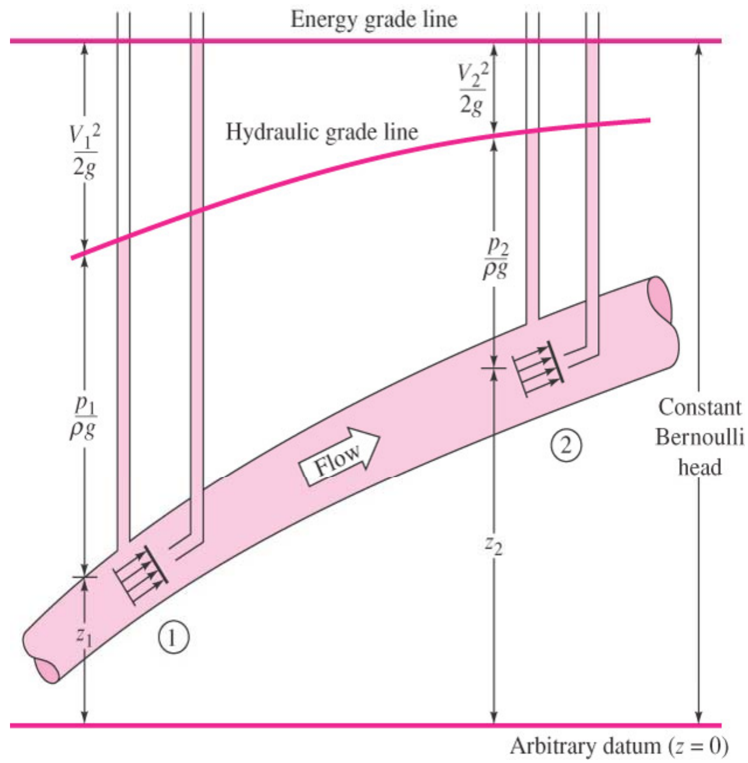
$$\left(\frac{p_1}{\rho}+\frac{\alpha V_1^2}{2}+gz_1\right)=\left(\frac{p_2}{\rho}+\frac{\alpha V_2^2}{2}+gz_2\right)+(u_2-u_1-q)+w_s+w_v$$

$$\frac{p_1}{\rho}+\frac{1}{2}V_1^2+gz_1=\frac{p_2}{\rho}+\frac{1}{2}V_2^2+gz_2$$

- Note that the Bernoulli equation is a momentum-based force relation and is derived assuming:
- i) Steady flow
- ii) Incompressible flow (applicable for Mach numbers less than 0.3)
- iii) Frictionless flow (restrictive- where solid walls or mixing exist)
- iv) Flow along a single streamline.

Hydraulic and energy line

- The energy grade line (EGL) shows the height of the total Bernoulli constant, $h_0 = z + p/\gamma + V^2/(2g)$
- The hydraulic grade line (HGL) shows the height corresponding to elevation and pressure head $z + p/\gamma$ which is EGL minus the velocity head $V^2/(2g)$.



Example: Venturi tube

- A constriction in a pipe will cause the velocity to rise and the pressure to fall at section 2 in the throat. The pressure difference is a measure of the flow rate through the pipe. The smoothly necked-down system shown in figure below is called venturi tube.
- Find an expression for the mass flux in the tube as a function of the pressure change.