

Flow in viscous ducts

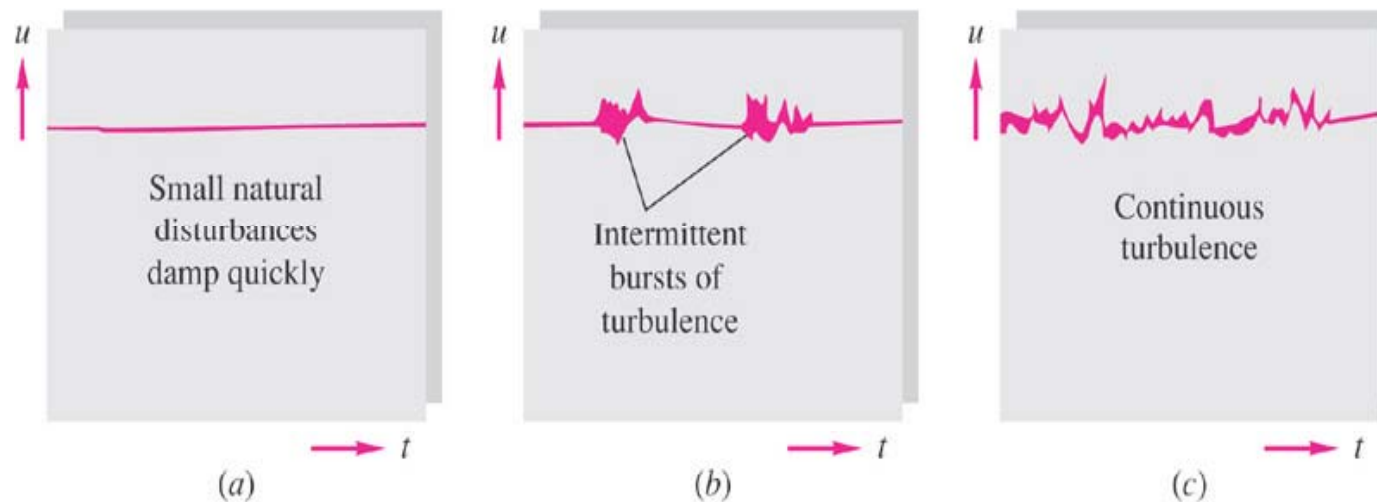
- We want to study the viscous flow in ducts with various velocities, fluids and duct shapes.
- The basic problem is this: Given the pipe geometry and its added components (e.g. fittings, valves, bends, and diffusers) plus the desired flow rate and fluid properties, what pressure drop is needed to drive the flow?

Reynolds number

The most important parameter in fluid mechanics is the Reynolds number:

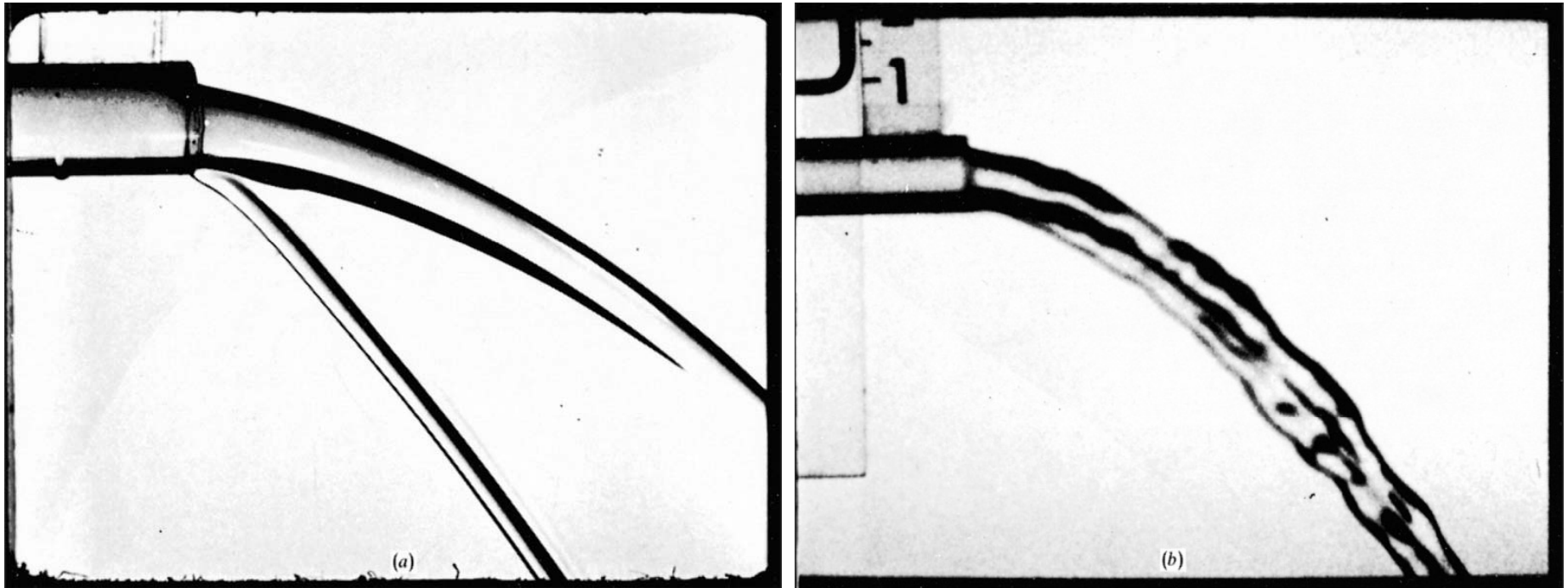
$$Re = \frac{\rho UL}{\mu}$$

The Reynolds number can be interpreted as the ratio of momentum (or inertia) to viscous forces.



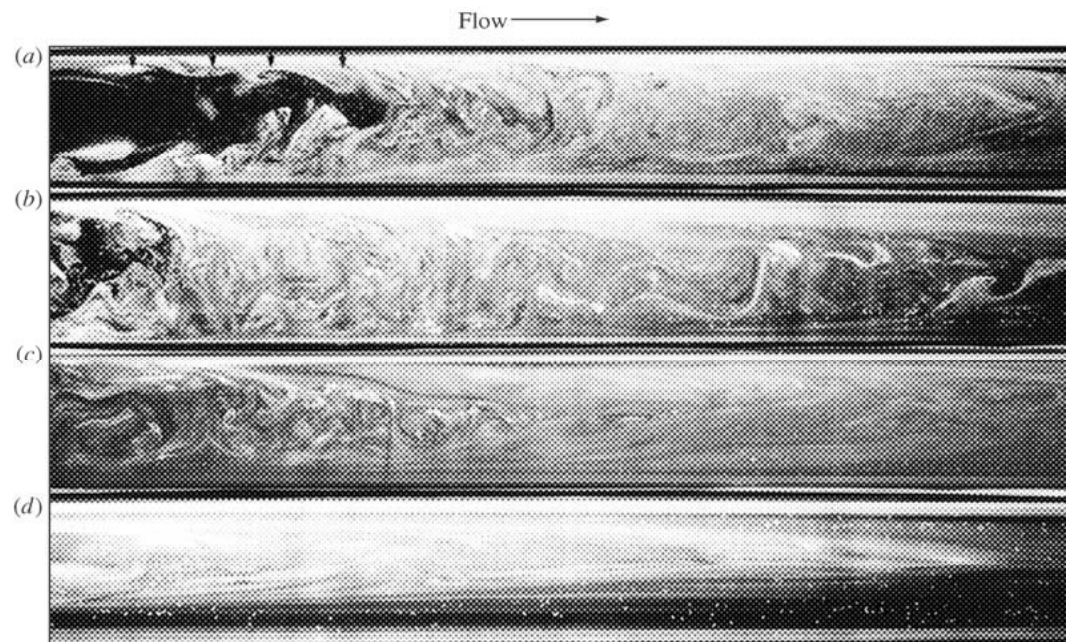
Laminar and turbulent regimes

A profound change in fluid behavior occurs at moderate Reynolds number. The flow ceases being smooth and steady (laminar) and becomes fluctuating (turbulent). The changeover is called transition.



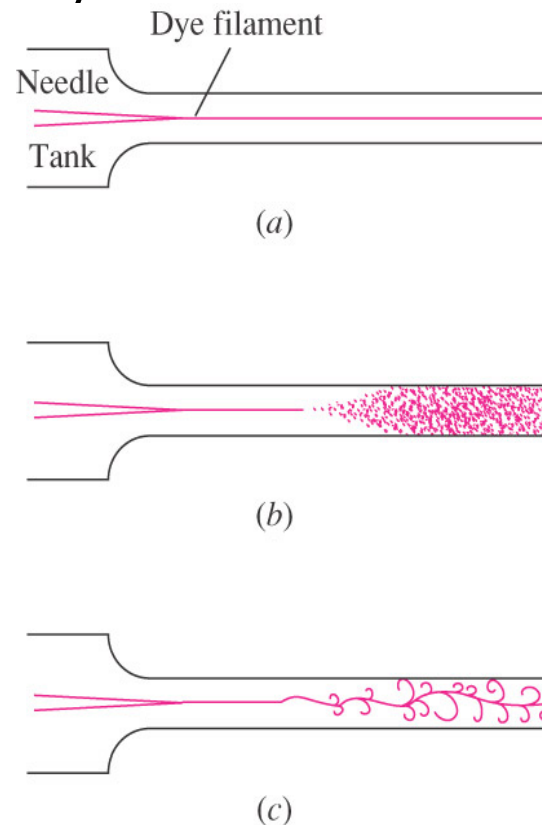
Reynolds number regimes

- $0 < Re < 1$: highly viscous laminar “creeping” motion
- $1 < Re < 100$: laminar strong Re dependence
- $100 < Re < 10^3$: laminar, boundary layer theory useful
- $10^3 < Re < 10^4$: transition to turbulence
- $10^4 < Re < 10^6$: turbulent, moderate Re dependence
- $10^6 < Re < \infty$: Turbulent, slight Re dependence



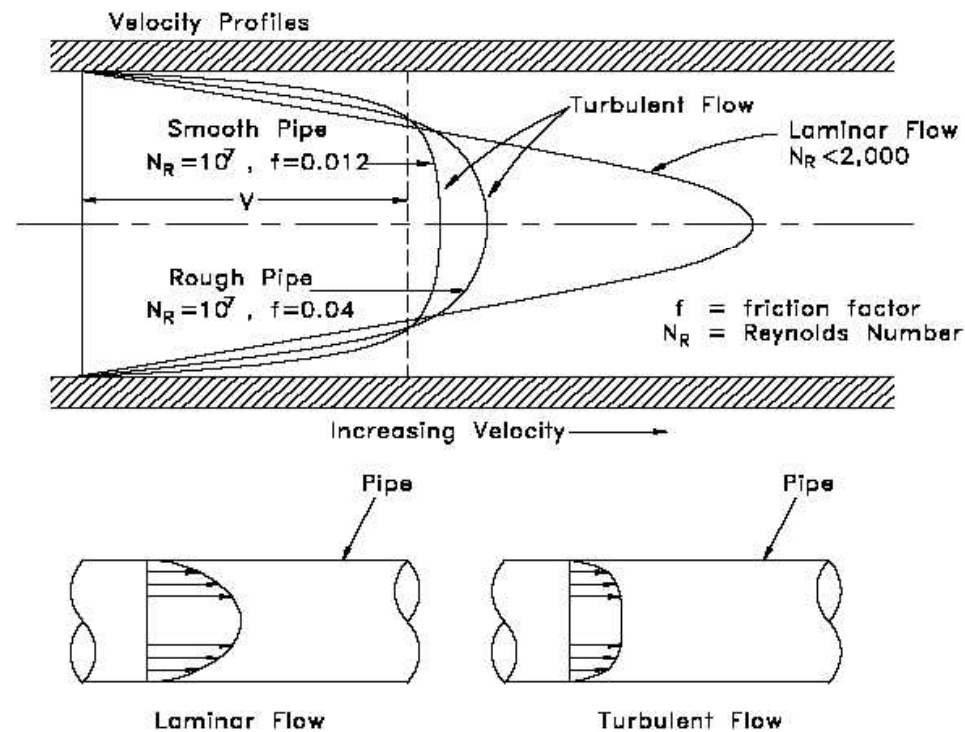
Reynolds experiment

- In 1883 Osborne Reynolds, a British engineering professor, observed the transition to turbulence in a pipe flow by introducing a dye streak in the flow.
- Reynolds experimentally showed that the transition occurs in a pipe flow at:



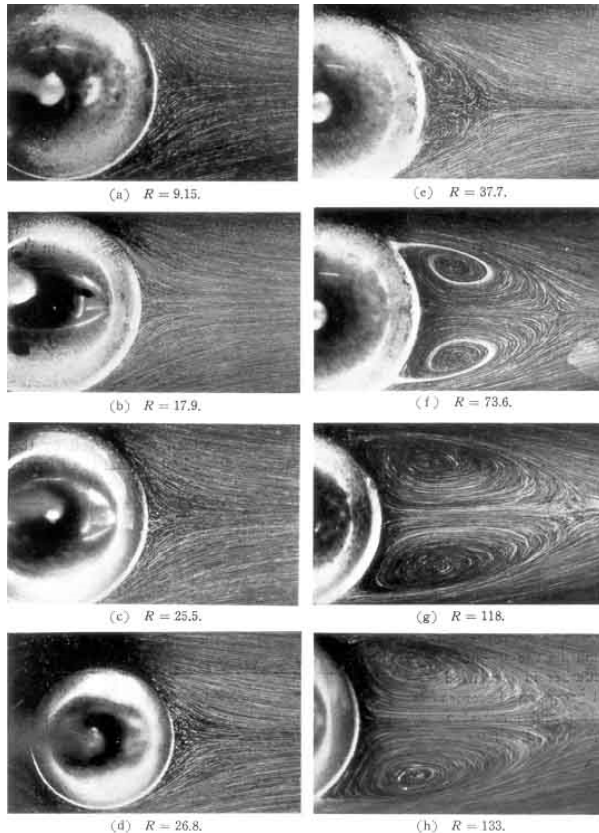
Internal viscous flow

- Both laminar and turbulent flows can be internal and external
- An internal flow is contained (or bounded) by walls and the viscous flow will grow and meet and permeate the entire flow. As a result, there is an entrance region where nearly inviscid upstream flow converges and enters the duct.



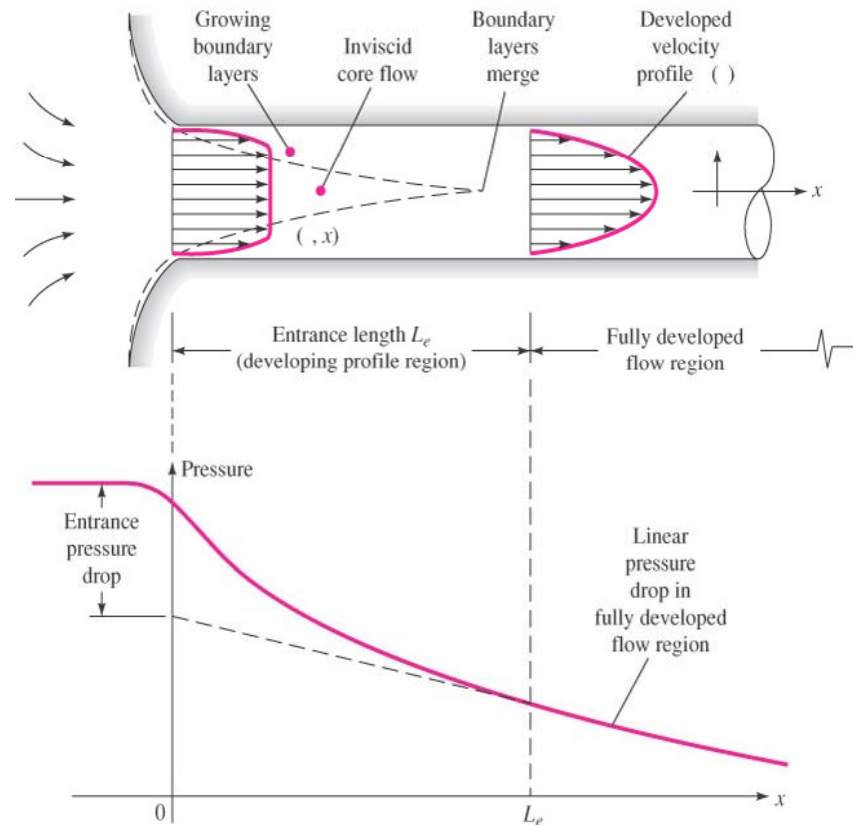
External viscous flow

- External flow has no restraining walls and is free to expand no matter how thick the viscous layers on the immersed body may become. As a result, far from the body the flow is nearly inviscid and there is no external equivalent of fully developed internal flow.



Entrance region

- Beyond the entrance region, which is a finite distance from the entrance $x = L_e$, the velocity profile becomes constant, i.e. it no longer changes with x and is said to be fully developed, $u \approx u(r)$.



Entrance region cont'd.

- For laminar flow, the entrance region can be found from the following empirical correlation:

$$\frac{L_e}{d} = 0.06Re_d \quad (\text{Laminar})$$

- Assuming the maximum Re for the laminar flow in a duct is $Re_{d,crit} = 2300$ the longest laminar developing region becomes: $L_e = 138d$, i.e. 138 times of the tube diameter.
- Turbulent flow boundary layers grow faster, and the entrance region (L_e) is relatively shorter:

$$\frac{L_e}{d} = 4.4Re_d^{1/6} \quad (\text{Turbulent})$$

- the entrance region length can vary from $40d$ to $100d$ for turbulent and laminar regimes, respectively. However, for typical pipe flow application, the pipe length is 1000 of its diameter in which case the entrance effect may be neglected.

Laminar friction factor

- Continuity equation:

$$Q_1 = Q_2 = \text{const} \quad \text{or} \quad V_1 = V_2 = V$$

- The energy equation:

$$\left(\frac{p}{\gamma} + \frac{V^2}{2g} + z \right)_1 = \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z \right)_2 + h_f$$

- Since $V_1 = V_2$, the friction head lost is:

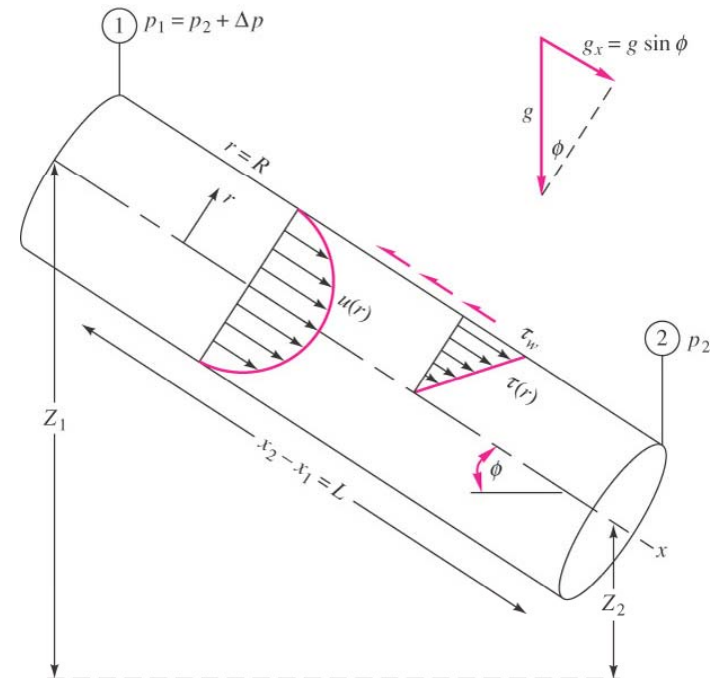
$$h_f = (z_1 - z_2) + \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g} \right) = \Delta z + \frac{\Delta p}{\rho g}$$

- Momentum along the x-direction:

$$\sum F_x = \Delta p (\pi R^2) + \rho g (\pi R^2) L \sin \phi - \tau_w (2\pi R) L = \dot{m} (V_2 - V_1) = 0$$

- Rearranging this, we find that the head loss is related to wall shear stress:

$$\Delta z + \frac{\Delta p}{\rho g} = h_f = \frac{2\tau_w L}{\rho g R} = \frac{4\tau_w L}{\rho g d}$$



Darcy friction factor

- A German professor, Julius Weisbach 1850, argued that the friction factor is proportional to L/d and V^2 (observed experimentally in turbulent regime). He then proposed to represent the frictionless head loss with a dimensionless parameter f (called the Darcy friction factor), that is defined as:

$$h_f = f \frac{LV^2}{d2g}$$

- By equating the above equations, we find:

$$f = \frac{8\tau_w}{\rho V^2}$$

$$f = f(Re_d, \text{duct roughness, duct shape})$$

The analytical solution for velocity in a round duct of diameter d is:

$$u(r) = u_{\max} \left(1 - \frac{r^2}{R^2} \right) \quad \text{where } u_{\max} = \left(-\frac{dp}{dx} \right) \frac{R^2}{4\mu} \quad \text{and} \quad \left(-\frac{dp}{dx} \right) = \frac{(\Delta p + \rho g \Delta z)}{L}$$

$$V = \frac{u_{\max}}{2} = \frac{(\Delta p + \rho g \Delta z) R^2}{8\mu} \quad \text{and} \quad Q = \int u dA = \pi R^2 V = \frac{\pi R^4}{8\mu} \left(\frac{\Delta p + \rho g \Delta z}{L} \right)$$

$$\tau_w = \left| \mu \frac{du}{dr} \right|_{r=R} = \frac{4\mu V}{R} = \frac{R}{2} \left(\frac{\Delta p + \rho g \Delta z}{L} \right)$$

$$h_f = \frac{32\mu LV}{\rho g d^2} = \frac{128\mu L Q}{\pi \rho g d^4}$$

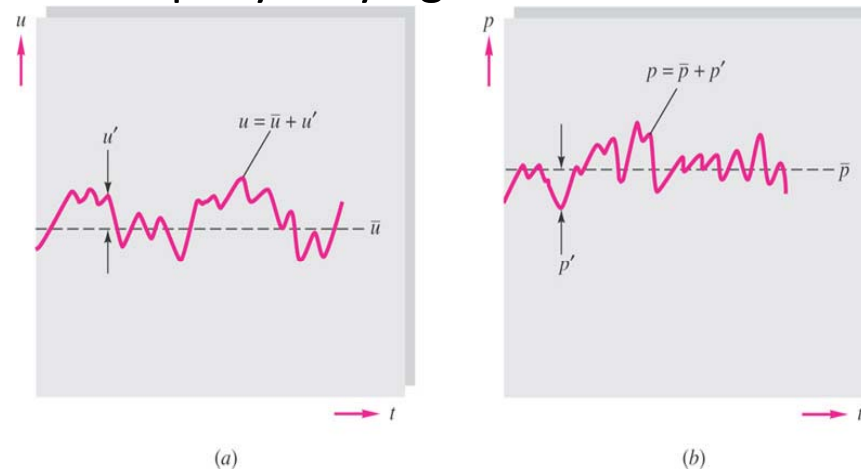
The pressure drop is inversely proportional to the pipe diameter to the power 4. So, if the size of the pipe is doubled, the pressure drop will decrease by a factor of 16 for a given Q .

Knowing the shear stress, the Poiseuille friction factor is easily determined:

$$f_{\text{laminar}} = \frac{8\tau_{w,\text{lam}}}{\rho V^2} = \frac{64}{Re_d}$$

Turbulent modeling

- For turbulent flow, every velocity and pressure term in momentum and energy equations is a rapidly varying random function of time and space.



- We are interested in the average or mean values in turbulent flow. Reynolds proposed to rewrite the governing equations in terms of average or mean values, defined as:

$$\bar{u} = \frac{1}{T} \int_0^T u dT$$

- The fluctuation u' is defined as the deviation of u from its average value:

$$u' = u - \bar{u}$$

Turbulent modeling cont'd.

- Using the concept of mean values, the momentum equation in x-direction (as an example) becomes:

$$\rho \frac{d\bar{u}}{dt} = \rho g_x - \frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x} \left(\mu \frac{\partial \bar{u}}{\partial x} - \rho \overline{u'^2} \right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'} \right) + \frac{\partial}{\partial z} \left(\mu \frac{\partial \bar{u}}{\partial z} - \rho \overline{u'w'} \right)$$

$-\rho \overline{u'^2}$, $-\rho \overline{u'v'}$, $-\rho \overline{u'w'}$ are called turbulent stresses because they have the same dimensions and occur right alongside the newtonian (laminar) stress terms.

Actually, they are convective acceleration terms (that is why the density appears), not stresses but they have the mathematical effect of stress.

Turbulent flow in pipes

There are three regions in turbulent flow, inner layer (or viscous sub-layer) where viscous effects are dominant (near the wall), overlap layer (transition to turbulent occurs) and outer region (the flow is completely turbulent)

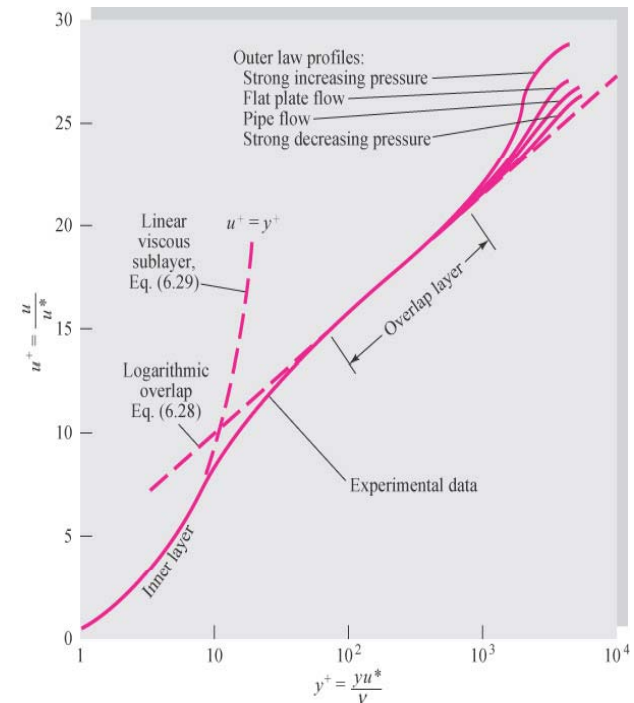
we only consider the overlap layer velocity profile for the entire pipe flow:

$$\frac{u(r)}{u^*} = \frac{1}{\kappa} \ln \frac{(R-r)u^*}{\nu} + B$$

where $\kappa=0.4$ and $B=5.0$, $\nu=\mu/\rho$ is the kinematic viscosity of the fluid, and

$$u^* = \left(\frac{\tau_w}{\rho} \right)^{1/2}$$

u^* is called the frictional velocity, because it has velocity dimensions, although it is not actually a flow velocity.



Friction factor for turbulent flow

The friction factor for the turbulent pipe flow can be calculated from the following correlations:

$$f = \begin{cases} 0.316 Re_d^{-1/4} & 4000 \leq Re_d \leq 10^5 \\ \left(1.8 \log \frac{Re_d}{6.9}\right)^{-2} & \end{cases}$$

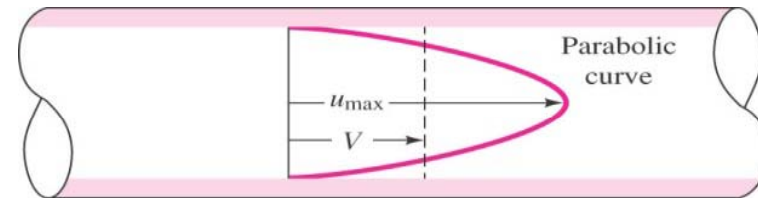
$$Re_d = Vd/\nu$$

For a horizontal pipe, we have:

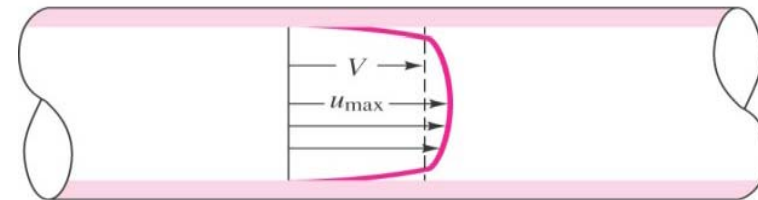
$$\Delta p = 0.158 L \rho^{3/4} \mu^{1/4} d^{-5/4} V^{7/4}$$

$$\Delta p = 0.24 L \rho^{3/4} \mu^{1/4} d^{-4.75} Q^{1.75}$$

The pressure drop for the turbulent flow decreases with diameter even more sharply than the laminar flow. Doubling the pipe size decreases the pressure drop by a factor of 27 for a given Q.



(a)

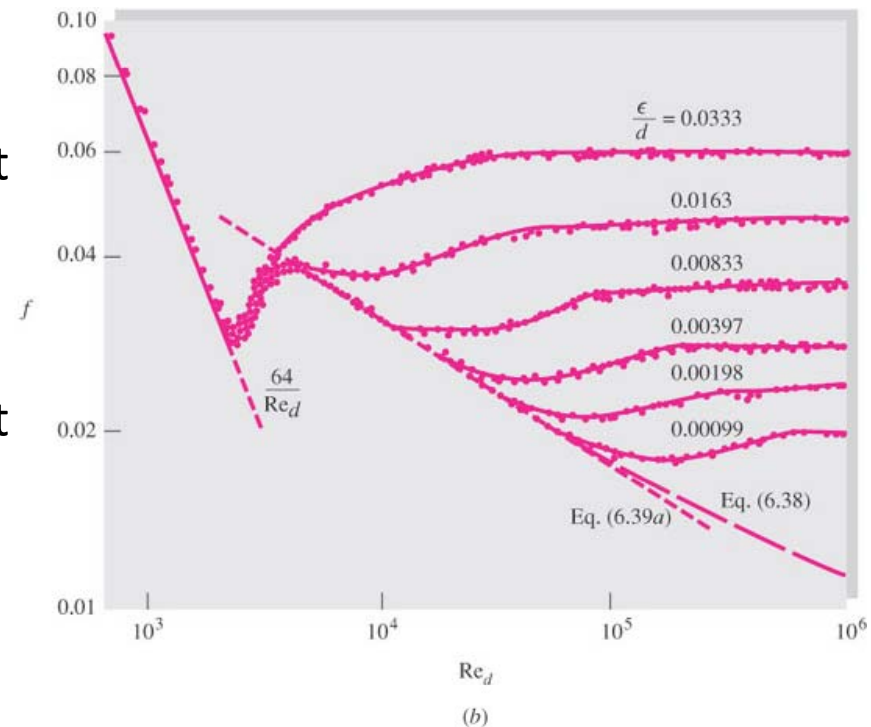


(b)

Effect of wall roughness

The surface roughness has an effect on friction resistance; for laminar flow, however, this effect is negligible. The turbulent flow is strongly affected by roughness. Nikuradse (1933) simulated roughness by gluing uniform sand grains onto the inner walls of the pipes.

- The laminar friction is unaffected
- The turbulent friction, after an onset point, increases monotonically with the roughness ratio, ϵ/d
- The friction factor becomes constant (fully rough) at high Reynolds numbers.



Flow in rough pipes

The logarithm law modified for roughness becomes:

$$u^+ = \frac{u}{u^*} = \frac{1}{\kappa} \ln\left(\frac{y}{\epsilon}\right) + 8.5$$

Integrating this equation, we can find the average velocity in the pipe:

$$\frac{V}{u^*} = 2.44 \ln\left(\frac{d}{\epsilon}\right) + 3.2$$

For fully rough flow, we have:

$$\frac{1}{f^{1/2}} = -2.0 \log\left(\frac{\epsilon/d}{3.7}\right)$$

Notice that there is no Reynolds number effect; hence the head loss varies exactly as the square of the velocity in this case.

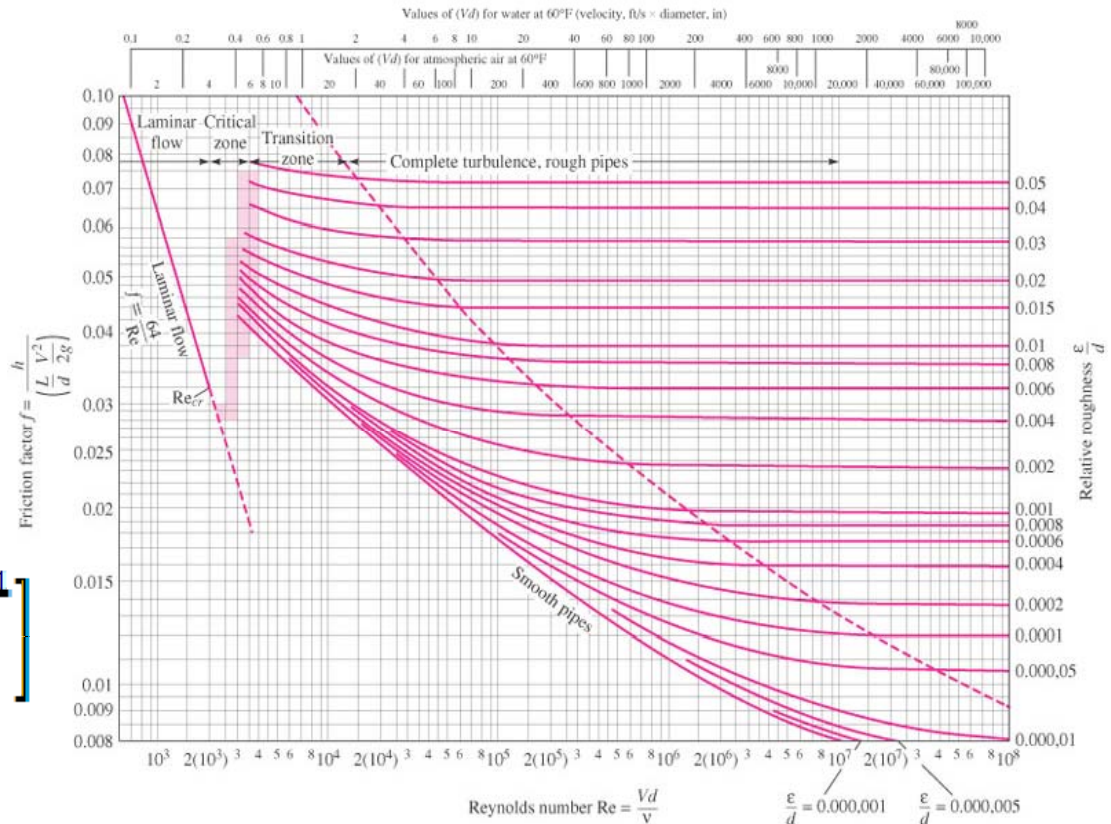
The Moody chart

Colebrook (1939) combined the smooth wall and fully rough relations into a clever combined interpolation formula (with 15% accuracy):

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\epsilon/d}{3.7} + \frac{2.51}{Re_d \sqrt{f}} \right)$$

Haaland (1983) proposed another correlation:

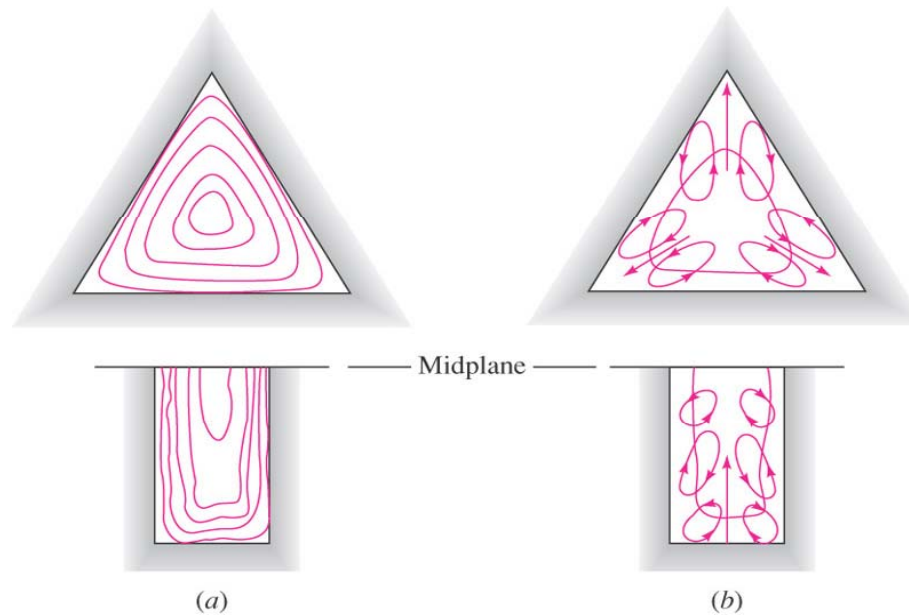
$$\frac{1}{\sqrt{f}} = -1.8 \log \left[\frac{6.9}{Re_d} + \left(\frac{\epsilon/d}{3.7} \right)^{1.11} \right]$$



Hydraulic diameter

In general, for complex geometries, it is very challenging to perform the flow analysis directly. Therefore, the concept of the hydraulic diameter is introduced for convenience.

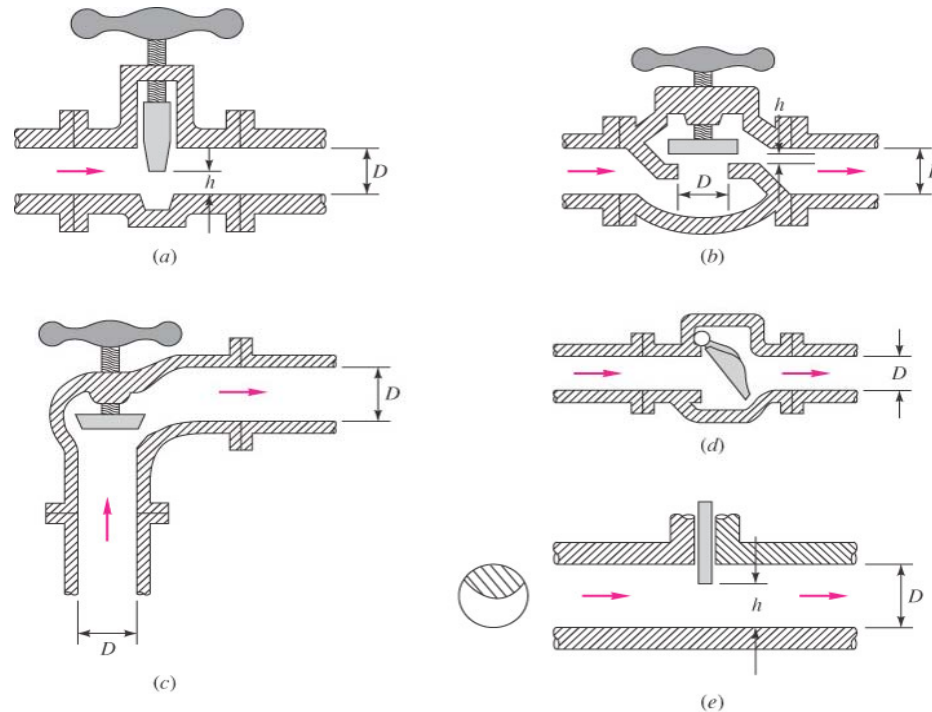
$$D_h = \frac{4 \times \text{area}}{\text{wetted perimeter}} = \frac{4A}{\phi}$$



We use the hydraulic diameter, as a length scale for non-circular ducts, and use the analyses derived for the circular pipes.

Minor losses in pipe systems

For any pipe systems, in addition to the Moody-type friction loss, computed for the length of pipe, there are additional minor losses, including: pipe entrance or exit, sudden expansion or contraction, bends, elbows, tees, other fittings, valves (open or partially closed), and gradual expansions or contractions.



Minor losses

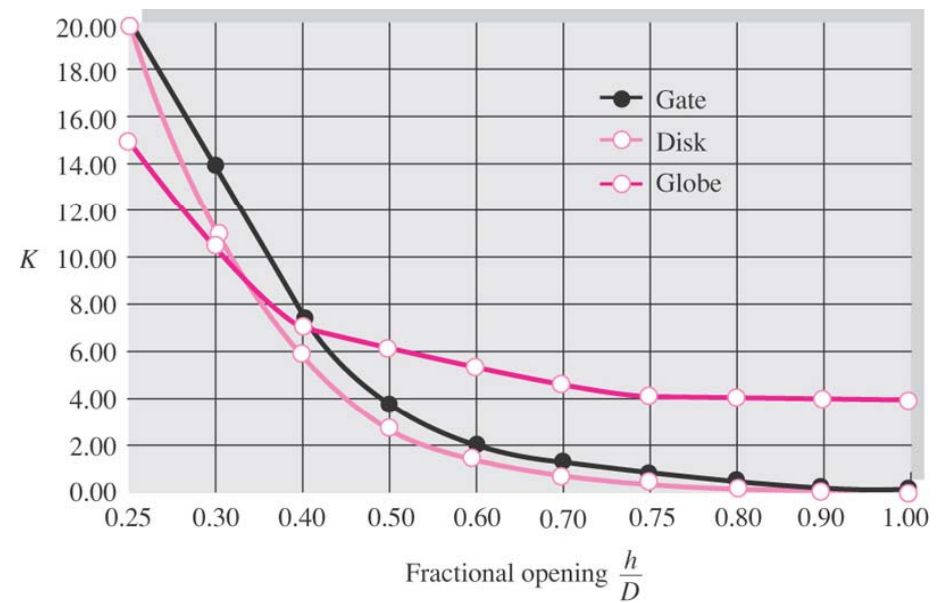
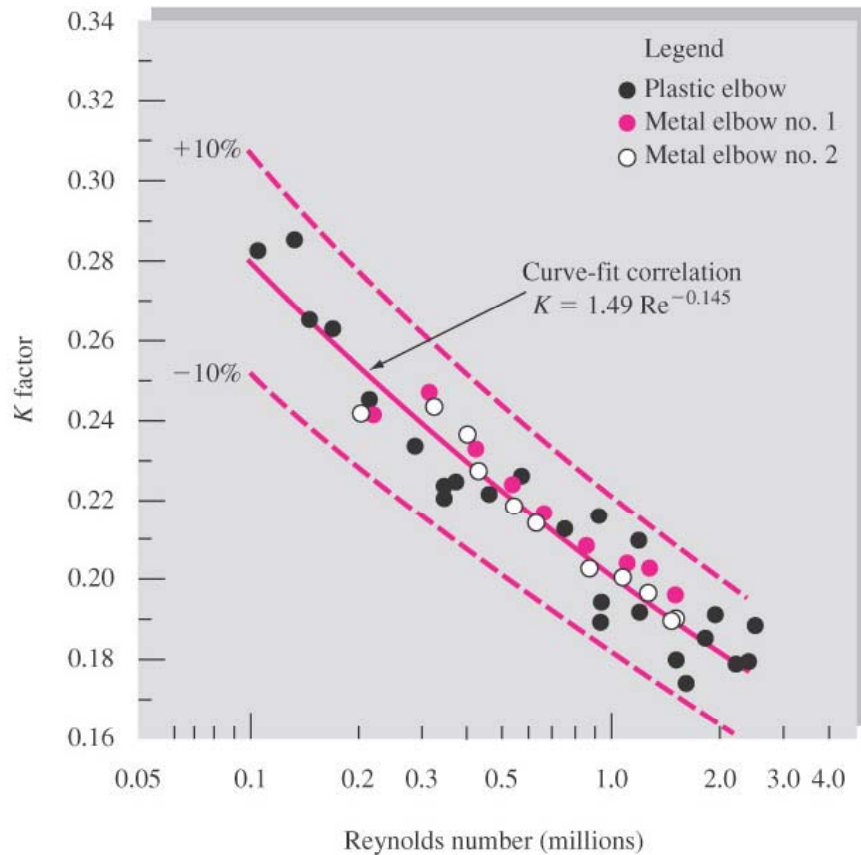
The losses commonly measured experimentally and correlated with the pipe flow parameters, usually given as a ratio of the head loss through the device to the velocity head of the associated piping system:

$$\text{Loss coefficient } K = \frac{h_m}{V^2/(2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

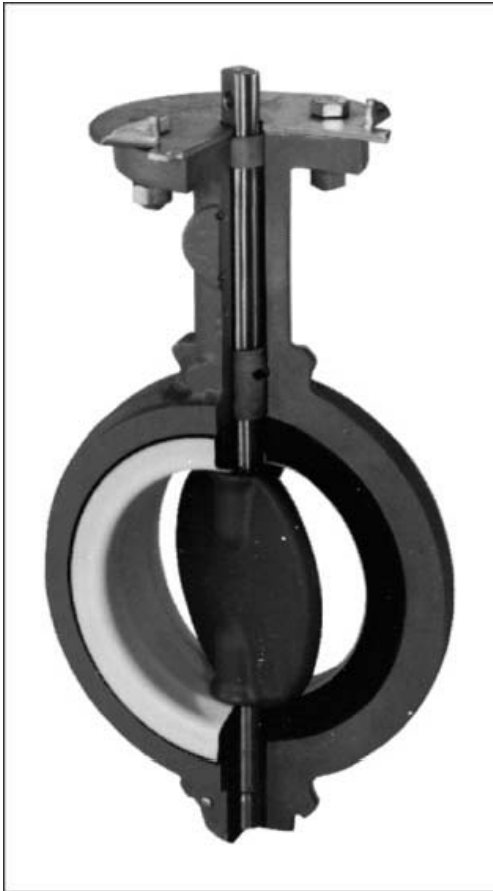
A single pipe system may have many minor losses. Since all are correlated with $V^2/(2g)$ they can be summed into a single total system loss if the pipe has constant diameter:

$$\Delta h_{\text{tot}} = h_f + \sum h_m = \frac{V^2}{2g} \left(f \frac{L}{d} + \sum K \right)$$

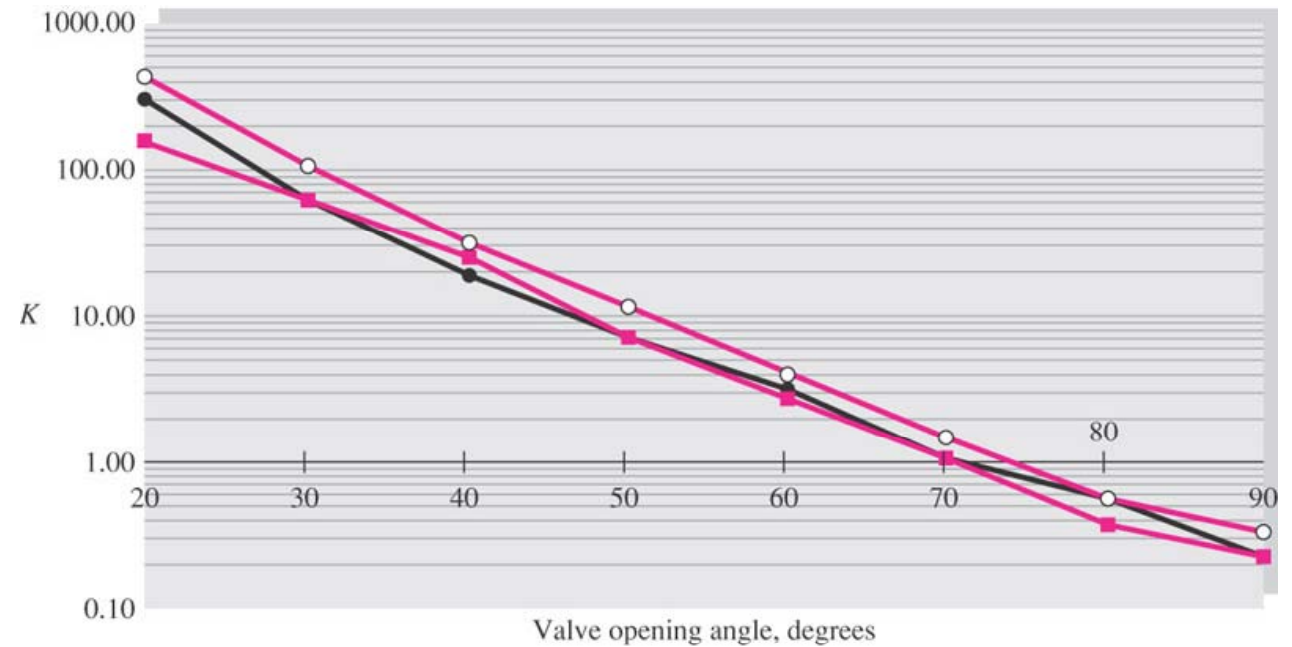
Some example for K factor



K factor for butterfly valve

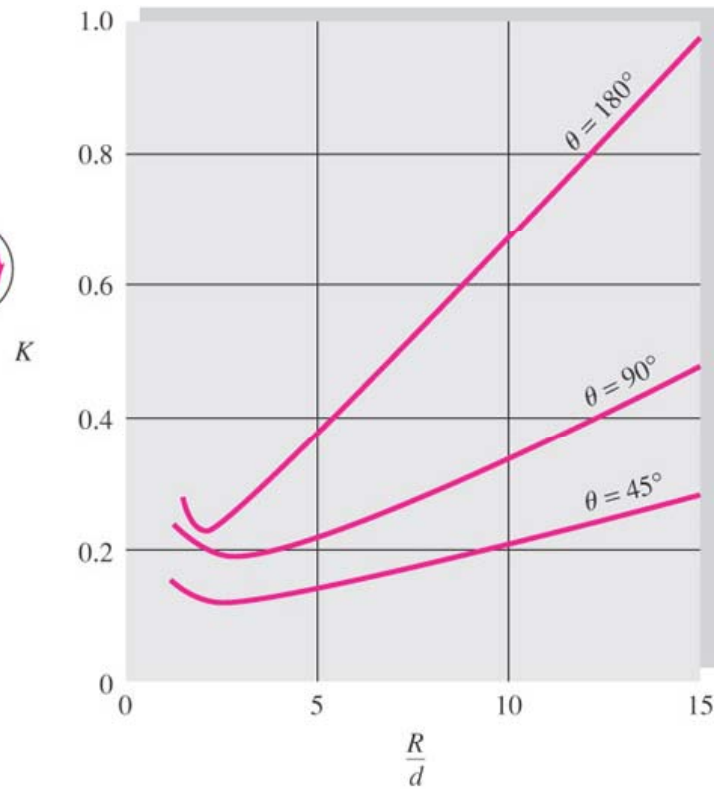
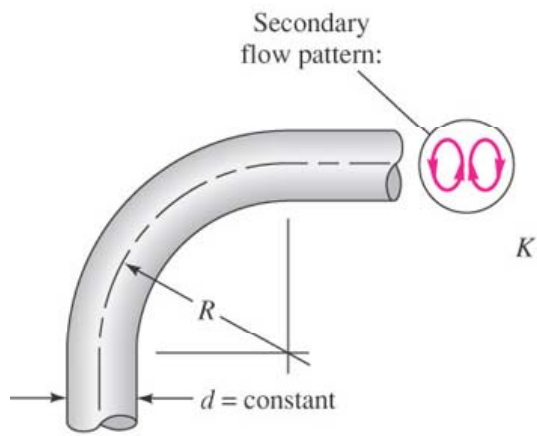


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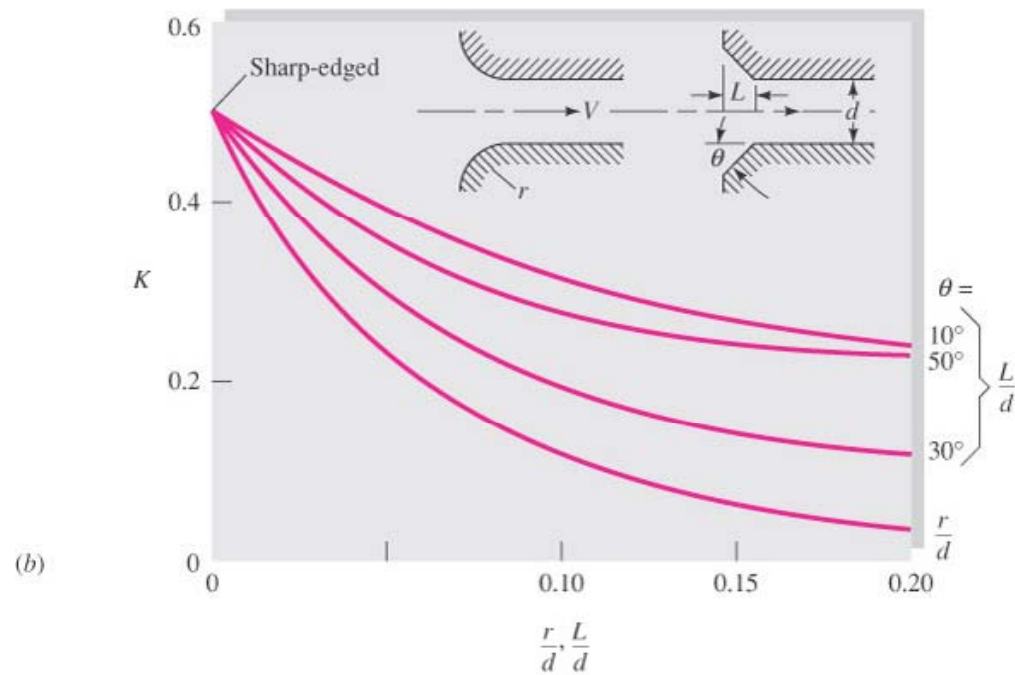


(b)

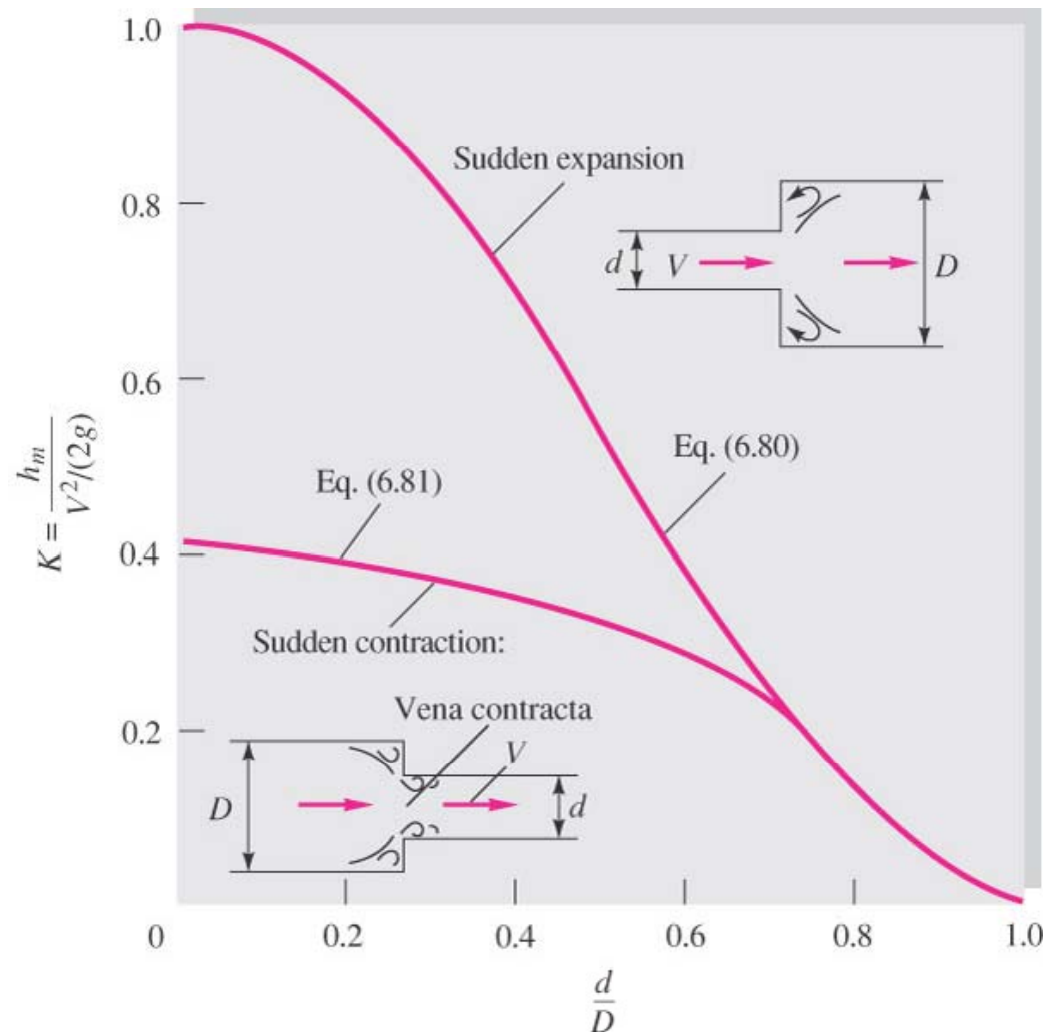
Bends K factor



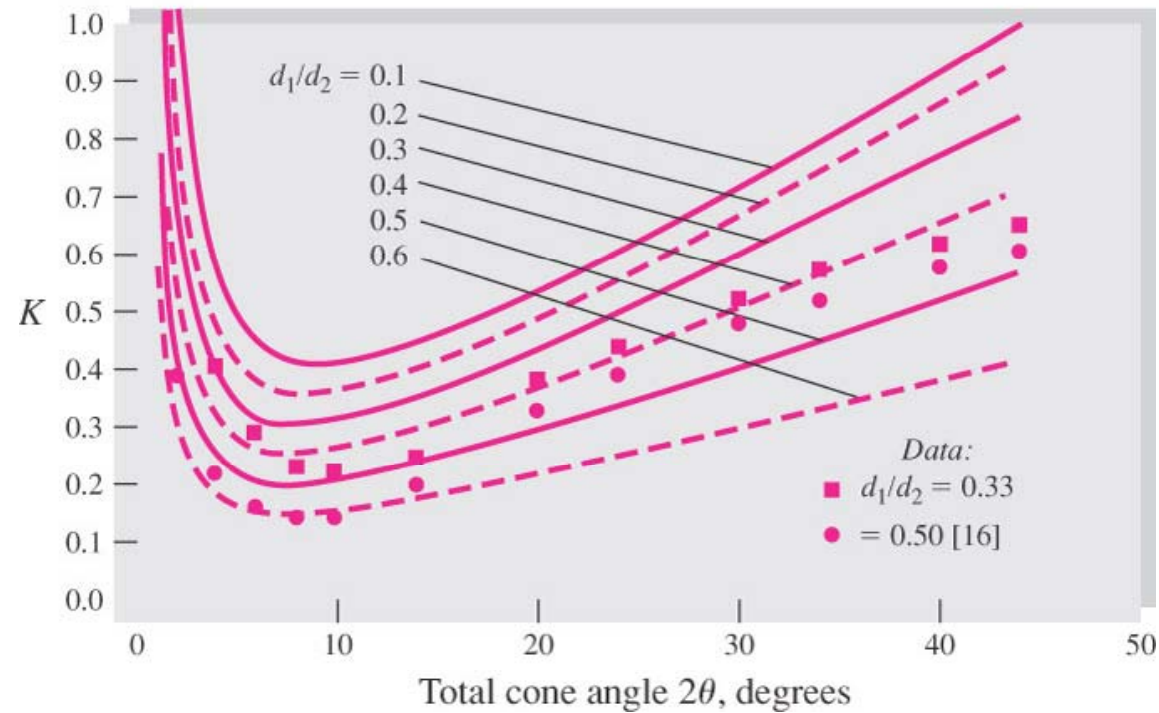
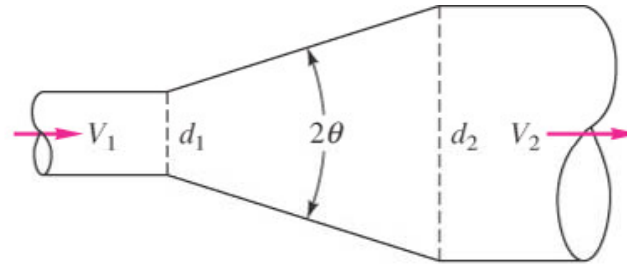
Entrance and exit K factor



Sudden expansion & contraction



Gradual conical expansion K



Flow meters

