Buoyancy

Archimedes's 1st laws of buoyancy: A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces, see Fig. 9 and 10.

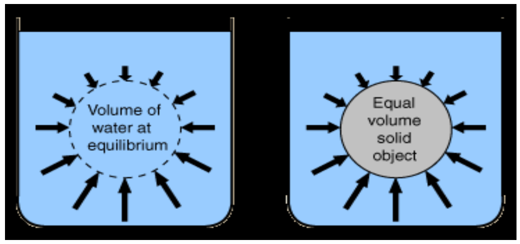


Fig. 9: an immersed body in a fluid, experiences a force equal to the weight of the fluid it displaces.

The line of action of the buoyant force passes through the center of volume of the displaced body; i.e., the center of mass is computed as if it had uniform density. The point which F_B acts is called the center of buoyancy.

Both liquids and gases exert buoyancy force on immersed bodies.

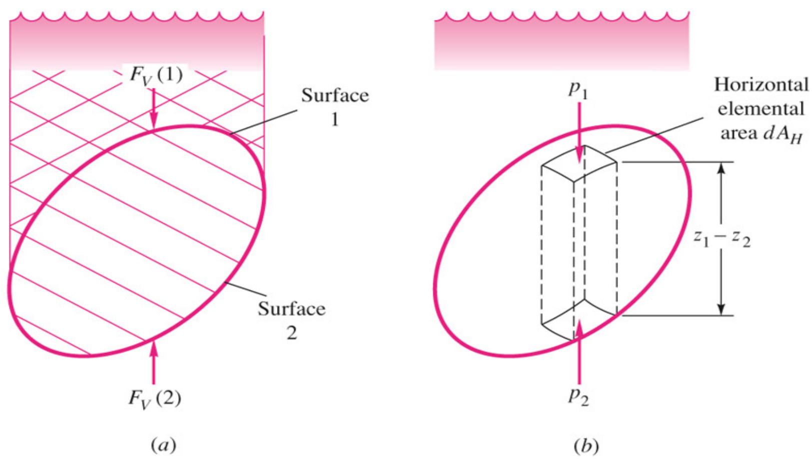


Fig.10: Archimedes first law of buoyancy.

$$F_B = \int_{body} (p_2 - p_1)\, dA_H = -\gamma \int (z_2 - z_1)\, dA_H = \gamma(body\ volume)$$

This equation assumes that the body has a uniform specific weight.

A floating body displaces its own weight in the fluid in which it floats.

In the case of a floating body, only a portion of the body is submerged, thus:

$$F_B = \gamma(displaced\ volume) = \text{weight of the floating body}$$

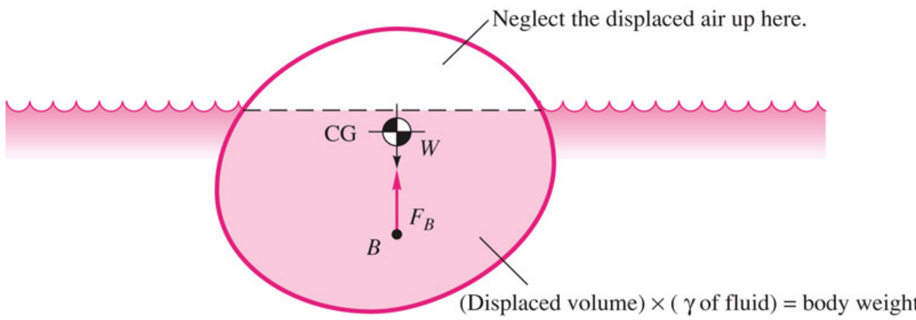


Fig. 11: Archimedes second law of buoyancy.

Example: Buoyancy force on a submerged object

A spherical body has a diameter of 1.5 m, weighs 8.5 kN, and is anchored to the sea floor with a cable as is shown in the figure. Calculate the tension of the cable when the body is completely immersed, assume $\gamma_{sea-water} = 10.1\ kN/m^3$.

Solution:

The buoyancy force F_B is shown in the free-body-diagram where W is the weight of the body and T is the cable tension. For equilibrium, we have:

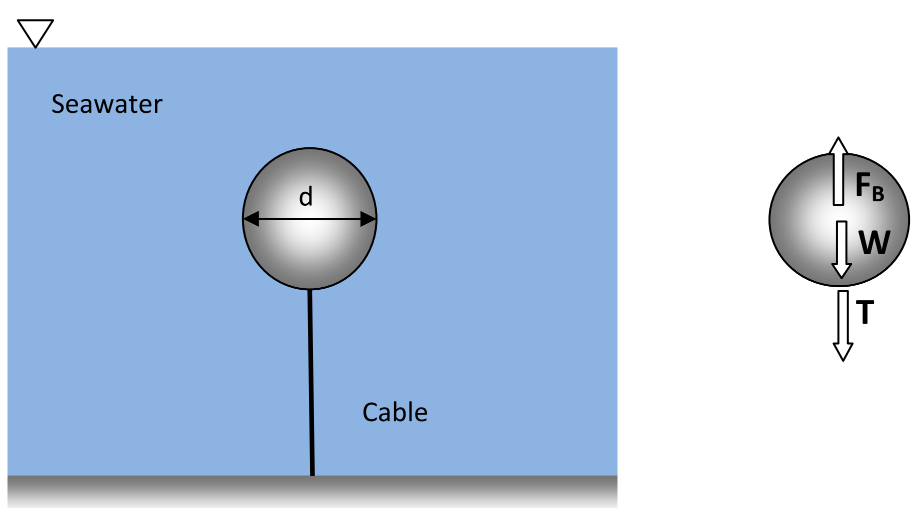
$$T = F_B - W$$

The buoyancy force is; $F_B = \gamma V$. And the volume of the body is:

$$V = \frac{\pi d^3}{6}$$

The cable tension then becomes:

$$T = 10.1 \times 10^3 (N/m^3) \frac{\pi[1.5(m)]^3}{6} - 8.50 \times 10^3 (N) = 9.35 kN$$



Pressure distribution in rigid-body motion

Fluids move in rigid-body motion only when restrained by confining walls. In rigid-body motion, all particles are in combined translation and rotation, and there is no relative motion between particles. The force balance equation becomes:

$$\nabla p = \rho(\vec{g} - \vec{a})$$

where a is the acceleration. The pressure gradient acts in the direction of $g - a$ and lines of constant pressure (including the free surface, if any) are perpendicular to this direction and thus tilted at a downward angle θ (see Fig. 11) such that:

$$\theta = tan^{-1}\frac{a_x}{g + a_z}$$

The rate of increase of pressure in the direction $g - a$ is greater than in ordinary hydrostatics:

$$\frac{dp}{ds} = \rho G \;\; where \;\; G = [a_x^2 + (g + a_z)^2]^{1/2}$$

Note: the results are independent of the size or shape of the container as long as the fluid is continuously connected throughout the container.

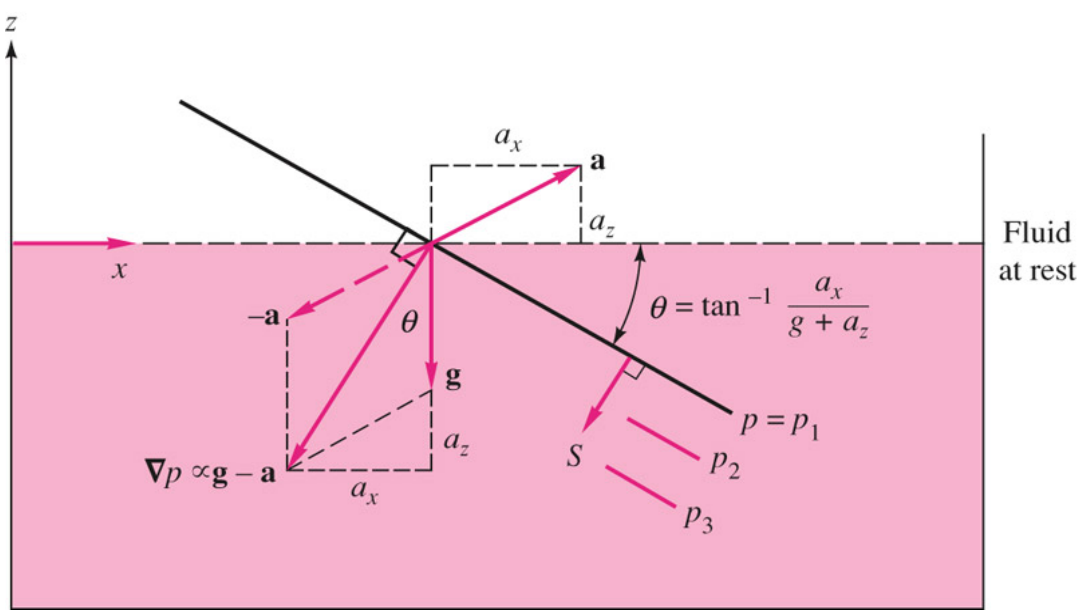


Fig.11: Rigid-body motion of a fluid contained in a tank.

Rigid-body rotation

Consider rotation of the fluid about the z-axis without any translation, Fig. 12. The container is assumed to be rotating at a constant angular velocity Ω for a long time.

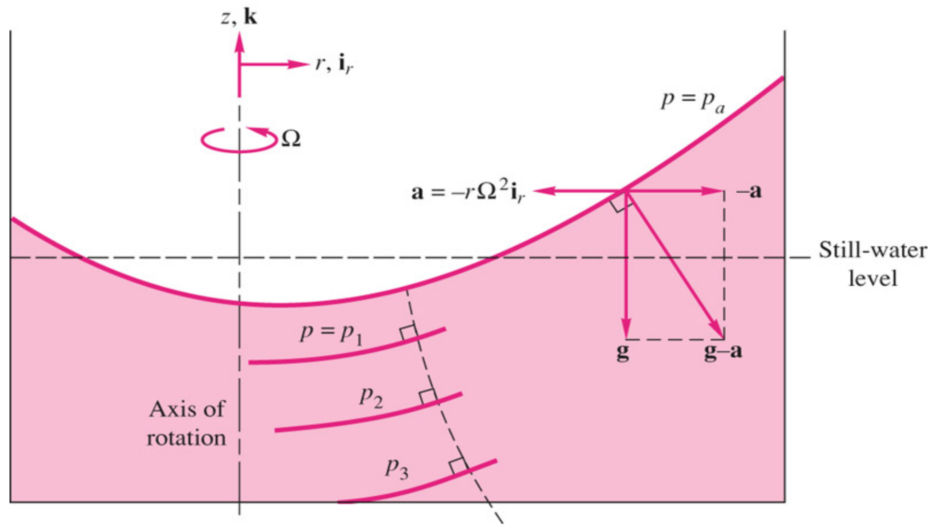


Fig. 12: Paraboloid constant-pressure surfaces in a fluid in rigid-body rotation.

The angular velocity and position vectors are given by:

$$\vec{\Omega} = \Omega k \qquad \vec{r} = r i_r$$

The acceleration id given by:

$$\Omega \times (\Omega \times \vec{r}) = -r\Omega^2 i_r$$

The forced balance becomes:

$$\nabla p = \frac{\partial p}{\partial r} i_r + \frac{\partial p}{\partial z} k = \rho(g-a) = \rho(-gk + r\Omega^2 i_r)$$

The pressure field can be found by equating like components:

$$\frac{\partial p}{\partial r} = \rho r \Omega^2 \qquad \frac{\partial p}{\partial z} = -\gamma$$

After integration with respect to r and z, and applying boundary condition, p=p_0 at (r,z) = (0,0):

$$p = p_0 - \gamma z + \frac{1}{2}\rho r^2 \Omega^2$$

The pressure is linear in z and parabolic in r. The constant pressure surfaces can be calculated using:

$$z = \frac{p_0 - p_1}{\gamma} + \frac{r^2\Omega^2}{2g} = a + br^2$$

The position of the free surface is found by conserving the volume of fluid. Since the volume of a paraboloid is one-half the base area times its height, the still water level is exactly halfway between the high and low points of the free surface.

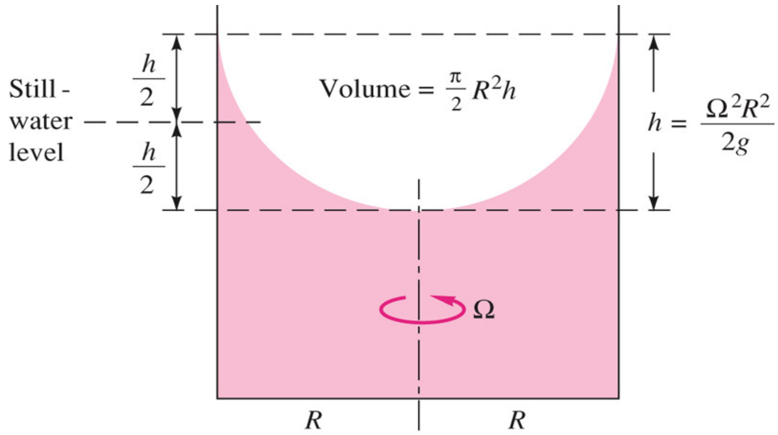


Fig. 13: Determining the free-surface position for rotation of a cylinder of fluid about its axis. The center of the fluid drops an amount $h/2 = \Omega^2 R^2/4g$, and edges rise an equal amount.

Pressure measurement

Pressure is the force per unit area and can be imagined as the effects related to fluid molecular bombardment of a surface. There are many devices for both a static fluid and moving fluid pressure measurements. Manometer, barometer, Bourdon gage, McLeod gage, Knudsen gage are only a few examples.

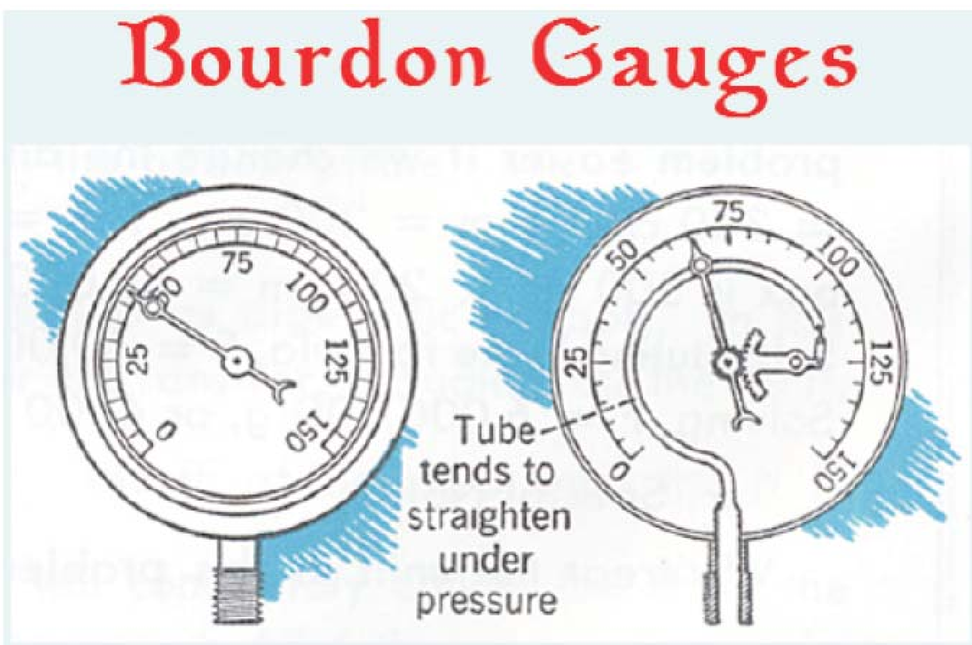


Fig. 14: Bourdon gage.