ENSC283 INTRODUCTION TO FLUID MECHANICS

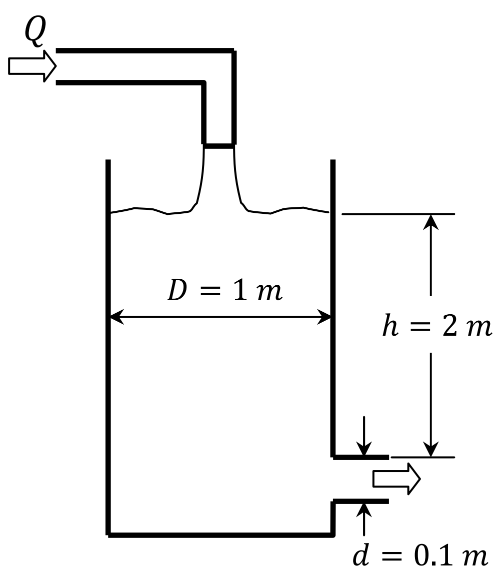
27 February 2009

Midterm Examination M. Bahrami

- This is a 2-1/2 hours, closed-book and notes examination.
- You are permitted to use one 8.5 in.× 11 in. crib sheet (double-sided) and the property tables.
- There are 5 questions to be answered. Read the questions very carefully.
- Clearly state all assumptions.
- It is your responsibility to write clearly and legibly.

<u>Problem 1:</u> (20 marks)

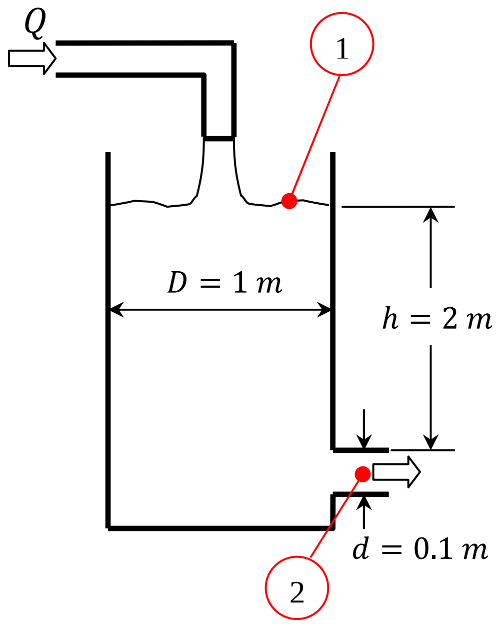
A stream of water of diameter $d = 0.1\ m$ flows steadily from a tank of diameter $D = 1\ m$ as shown in the figure. Determine the flow rate, Q, needed from the inflow pipe if the water depth remains constant, ($h = 2\ m$).



<u>Problem 1:</u> (Solution)

For steady, incompressible flow, the Bernoulli equation applied between points 1 and 2

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$



With the assumptions that $p_1 = p_2 = 0$, $z_1 = h$ and $z_2 = 0$,

$$\frac{1}{2}V_1^2 + gh = \frac{1}{2}V_2^2 \quad \text{(I)}$$

Although the water level remains constant, there is an average velocity, V_1, across section 1 because of the flow from the tank. For steady incompressible flow, conservation of mass requires $Q_1 = Q_2$, where $Q = AV$. Thus, $A_1V_1 = A_2V_2$ or

$$D^2V_1 = d^2V_2$$

Hence,

$$V_1 = \left(\frac{d}{D}\right)^2 V_2 \quad \text{(II)}$$

Combining equations (I) and (II)

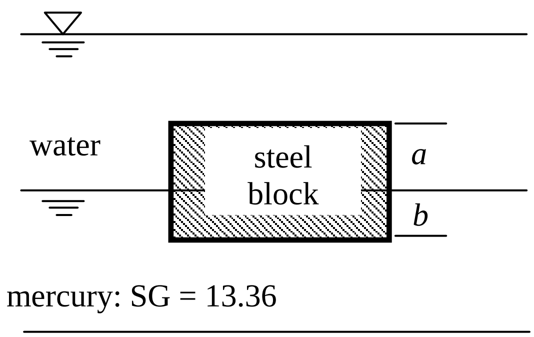
$$V_2 = \sqrt{\frac{2gh}{1 - \left(\frac{d}{D}\right)^4}} = \sqrt{\frac{2\left(9.81\ \left[\frac{m}{s^2}\right]\right)(2\ [m])}{1 - \left(\frac{0.1[m]}{1\ [m]}\right)^4}} = 6.26\ \left[\frac{m}{s}\right]$$

Thus,

$$Q = A_1 V_1 = \frac{\pi(0.1\ [m])^2}{4} \times 6.26\ \left[\frac{m}{s}\right] = 0.0492\ \left[\frac{m^3}{s}\right]$$

<u>Problem 2:</u> (20 marks)

A uniform block of steel (SG = 7.85) will float at a mercury-water interface as shown in the figure. What is the ratio of the distance a and b.



<u>Problem 2:</u> (Solution)

Let L be the block length into the paper, W is the block width and let γ be the water specific weight. Then the vertical force balance on the block is

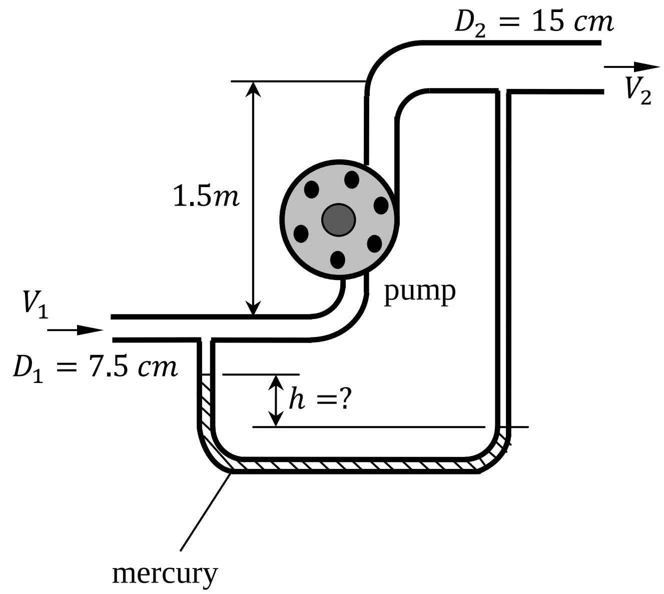
$$7.85\gamma(a+b)LW = 1\gamma aLW + 13.56\gamma bLW$$

Canceling W, L and γ from both sides of this equation and rearranging,

$$7.85a + 7.85b = a + 13.56b \rightarrow \frac{a}{b} = \frac{13.56 - 7.85}{7.85 - 1} = 0.834$$

Problem 3: **(20 marks)**

Kerosene at 20°C flows through the pump shown in figure at $0.065\ m^3/s$. Head losses between 1 and 2 are $2.4\ m$, and the pump delivers $6\ kW$ to the flow. What should the mercury-manometer reading h be? ($\gamma_{kerosene} = 8016.2\ N/m^3$ and $\gamma_{mercury} = 133210\ N/m^3$)



Problem 3: **(Solution)**

First establish two velocities, i.e. V_1 and V_2,

$$V_1 = \frac{Q}{A_1} = \frac{0.065\ \left[\frac{m^3}{s}\right]}{\left(\frac{\pi}{4}\right)(7.5 \times 0.01[m])^2} = 14.7\ \left[\frac{m}{s}\right]$$

$$\frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{D_1}{D_2}\right)^2 \rightarrow V_2 = \frac{1}{4}V_1 = 3.675\ \left[\frac{m}{s}\right]$$

To apply a manometer analysis to determine the pressure difference between points 1 and 2,

$p_2 - p_1 = (\gamma_m - \gamma_k)h - \gamma_k \Delta z = \left(133210\left[\frac{N}{m^3}\right] - 8016.2\left[\frac{N}{m^3}\right]\right)h - \left(8016.2\left[\frac{N}{m^3}\right]\right) \times 1.5[m] = 125193.8h - 12024.3\ \left[\frac{N}{m^2}\right]$

Now apply the steady flow energy equation between points 1 and 2

$$\frac{p_1}{\gamma_k} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_k} + \frac{V_2^2}{2g} + z_2 + h_f - h_p$$

where

$$h_p = \frac{P}{\gamma_k Q} = \frac{6000[W]}{8016.2\left[\frac{N}{m^3}\right] \times 0.068\left[\frac{m^3}{s}\right]} = 11\ [m]$$

Thus,

$$\frac{p_1}{8016.2\left[\frac{N}{m^3}\right]} + \frac{\left(14.7\ \left[\frac{m}{s}\right]\right)^2}{2(9.81)} + 0$$

$$= \frac{p_2}{8016.2\left[\frac{N}{m^3}\right]} + \frac{\left(3.675\ \left[\frac{m}{s}\right]\right)^2}{2(9.81)} + 1.5\ [m] + 2.4[m] - 11[m]$$

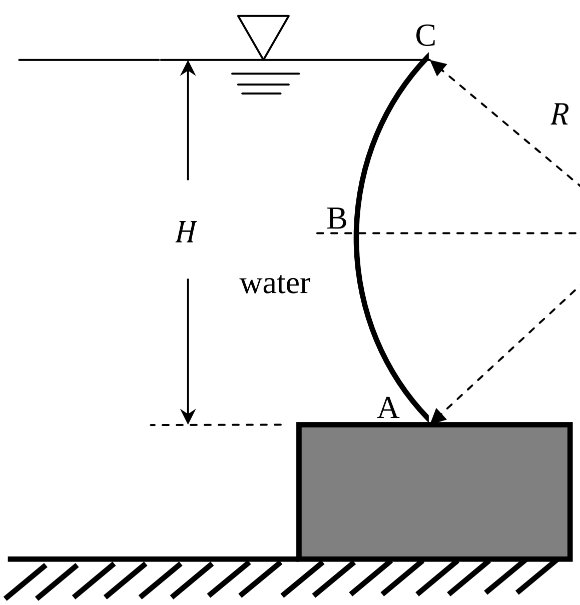
$$\rightarrow p_2 - p_1 = 139685\ \left[\frac{N}{m^2}\right] = 125193.8h - 12024.3\ \left[\frac{N}{m^2}\right]$$

or

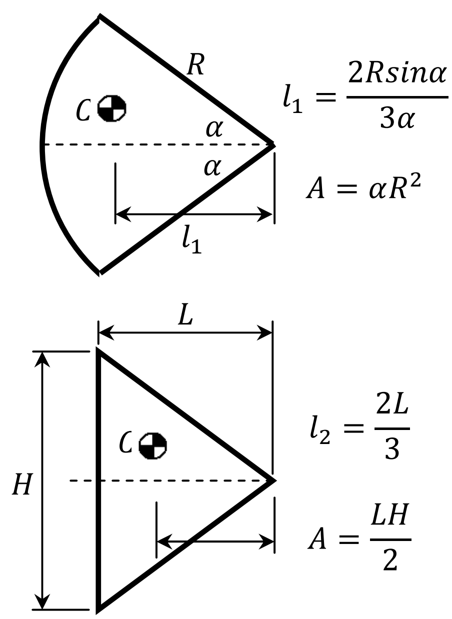
$$h = 1.211\ [m]$$

<u>Problem 4:</u> (20 marks)

Circular-arc gate ABC pivots about point O. For the position shown, determine (a) the hydrostatic force on the gate (per meter of width into the paper); and (b) its line of action.



Hint:

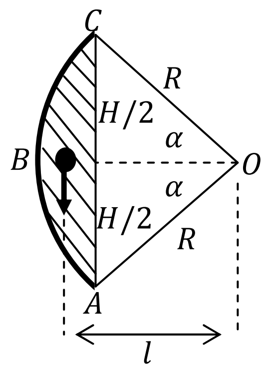


<u>Problem 4:</u> (Solution)

The horizontal hydrostatic force is based on the vertical projection:

$$F_H = \gamma_w h_{CG} A_{ver} = \frac{\gamma_w H^2}{2}$$

and the point of action for the horizontal force is 2H/3 below point C.
The vertical force is upward and equal to the weight of the missing water in the segment ABC shown hatched in the following figure.



The segment ABC area can be calculated from

$$A_{seg} = \alpha R^2 - \frac{RH\cos\alpha}{2}$$

This area gives the volume of water in segment ABC per unit width into the paper. Hence the water weight is,

$$F_V = \gamma_w A_{seg} = \gamma_w \left(\alpha R^2 - \frac{RH\cos\alpha}{2}\right)$$

To calculate the point of action, center of weight for segment ABC should be calculated. To do so, momentum balance around point O should be applied,

$$-\frac{HR\cos\alpha}{2} \times \frac{2R\cos\alpha}{3} + \alpha R^2 \times \frac{2R\sin\alpha}{3\alpha} = \left(\alpha R^2 - \frac{RH\cos\alpha}{2}\right) l$$

$$\rightarrow l = \frac{2R}{3}\left(\frac{2R\sin\alpha - H\cos^2\alpha}{2\alpha R - H\cos\alpha}\right)$$

The net force is thus

$$F = \sqrt{F_V^2 + F_H^2}$$

Per meter of width. This force is acting upward to the right at an angle of β where

$$\beta = tan^{-1}\left(\frac{H^2}{2\alpha R^2 - HRcos\alpha}\right)$$

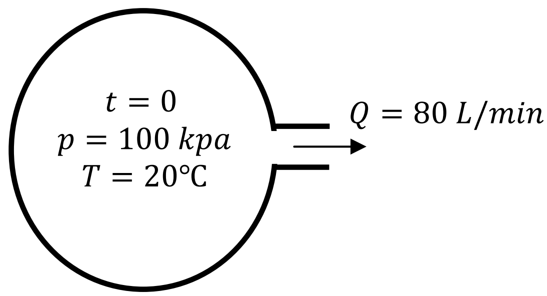
<u>Problem 5:</u> (20 mark)

A rigid tank of volume $V = 1m^3$ is initially filled with air at 20℃ and $p_0 = 100\, kpa$. At time $t = 0$, a vacuum pump is turned on and evacuates air at a constant volume flow rate of $Q = 80\, L/min$ (regardless of pressure). Assume an ideal gas and isothermal process.

(a) Set up a differential equation for this flow.

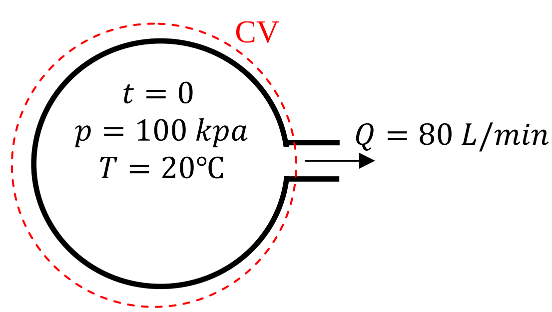
(b) Solve this equation for t as a function of (V, Q, p, p_0).

(c) Compute the time in minutes to pump the tank down to $p = 20\, kpa$.



<u>Problem 5:</u> (Solution)

The control volume encloses the tank, as shown is selected.



Continuity equation for the control volume becomes

$$\frac{d}{dt}\left(\int \rho dv\right) + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

Since no mass enters the control volume $\sum \dot{m}_{in} = 0$. Assuming that the gas density is uniform throughout the tank $\int \rho dv = \rho v$. Hence

$$v\frac{d\rho}{dt}+\rho Q=0$$

or

$$\int\frac{d\rho}{\rho}=-\frac{Q}{v}\int dt \quad \text{(part a)}$$

This differential equation can be easily solved yielding

$$ln\left(\frac{\rho}{\rho_0}\right)=-\frac{Qt}{v}$$

where ρ_0 is the gas initial density. For an isothermal ideal gas, $\rho/\rho_0 = p/p_0$, therefore

$$t=-\frac{v}{Q}ln\left(\frac{p}{p_0}\right) \text{(part b)}$$

For given values of $Q = 80L/min = 0.08\ m^3/min$, the time to pump a $1\ m^3$ tank down from 100 to 20 kpa is

$$t=-\frac{1[m^3]}{0.08\ [\frac{m^3}{min}]}ln\left(\frac{20}{100}\right)=20.1\ [min]$$