SFU

ENSC 283 Quiz #1

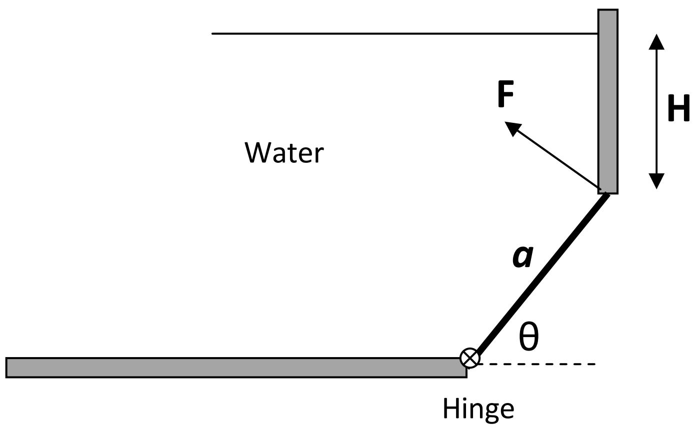
Jan. 27, 2009

Name: ……………………………………. Student ID:……………………………….

Time: 45 minutes or less. Develop answers on available place. The quiz has 5% (bonus) of the total mark. Closed books & closed notes.

Problem 1 (50%):

A square, side dimension *a* (m), has its top edge *H* (m) below the water surface. It is on angle θ and its bottom is hinged as shown in the figure below. Develop a relationship for the force F needed to just open the gate.



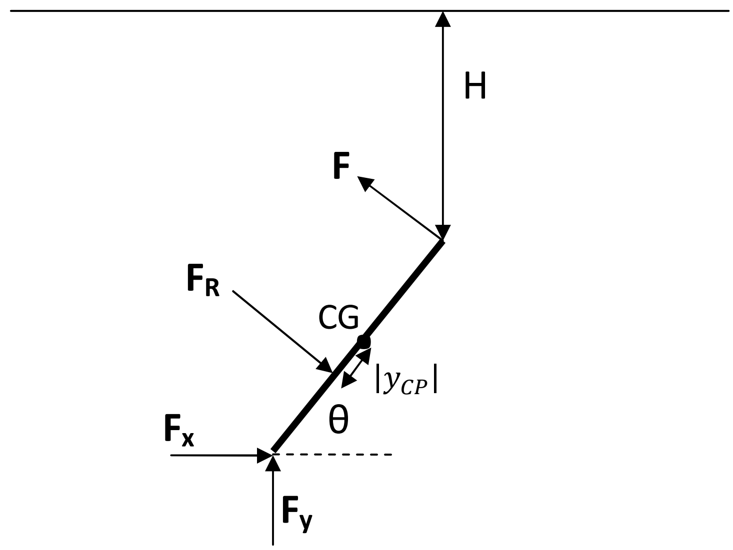
Hint: start with drawing a free-body-diagram of the gate. Also:

$$y_{CP} = -\gamma sin\theta \frac{I_{xx}}{p_{CG}A}$$

$$I_{xx} = \frac{a^4}{12} \qquad A = a^2$$

Solution:

The first step is to sketch a free-body diagram of the gate so the forces and distances are clearly identified. It is done in the following figure.



The force F_R is calculated to be

$$F_R = \gamma h_{CG} A = \gamma (H + \frac{a \sin\theta}{2}) a^2 \qquad \text{(Eq.1)}$$

We will take moments about the hinge so that it will not be necessary to calculate the forces F_x and F_y.

$$F_R \times \left[\frac{a}{2} - |y_{CP}|\right] = F \times a \qquad \text{(Eq.2)}$$

where, $|y_{CP}|$ is the distance between the center of pressure (CP) and the center of gravity (CG). $|y_{CP}|$ can be written as:

$$|y_{CP}| = \gamma sin\theta \frac{I_{xx}}{p_{CG} A} = \frac{I_{xx} \sin\theta}{h_{CG} A} = \frac{a^4}{12} \frac{\sin\theta}{(H + \frac{a \sin\theta}{2}) a^2} = \frac{a^2 \sin\theta}{12 (H + \frac{a \sin\theta}{2})} \qquad \text{(Eq.3)}$$

Substituting $|y_{CP}|$ into Eq.2, the force **F** is found.

$$F = \frac{F_R \times \left[\frac{a}{2} - |y_{CP}|\right]}{a} = \frac{\gamma(H + \frac{a\sin\theta}{2})a^2\left[\frac{a}{2} - |y_{CP}|\right]}{a} \qquad \text{(Eq.4)}$$

Simplifying the above equation, we get:

$$F = \frac{\gamma a^2(3H + a\sin\theta)}{6} \qquad \text{(Eq.5)}$$

Problem 2 (50%):

It is said that Archimedes discovered the buoyancy laws when asked by King Hiero of Syracuse to determine whether his new crown was pure gold (SG = 19.3). Archimedes measured the weight of the crown in air to be 11.8 N and its weight in water to be 10.9 N. Was it pure gold?

Hint: the buoyancy is the difference between air weight and underwater weight.

$$F_B = \gamma V$$

Solution:

The buoyancy is the difference between air weight and underwater weight:

$$F_B = W_{in\ air} - W_{in\ water} = \gamma_{water} V_{crown} = 11.8\ N - 10.9\ N = 0.9\ N \qquad \text{(Eq.1)}$$

where, $W_{in\ air}$ and $W_{in\ water}$ are the weight of the crown in air and water, respectively. The weight of the crown in air can be expressed as:

$$W_{in\ air} = (SG)\gamma_{water} V_{crown} \qquad \text{(Eq.2)}$$

Substituting Eq.2 into Eq.1, we get:

$$W_{in\ water} = \gamma_{water} V_{crown}(SG - 1) = F_B(SG - 1) \qquad \text{(Eq.3)}$$

Thus, the specific gravity of the crown can be written as:

$$SG = 1 + \frac{W_{in\ water}}{F_B} = 1 + \frac{10.9}{0.9} = \mathbf{13.11}\ (\boldsymbol{not\ pure\ gold}) \qquad \text{(Eq.4)}$$