ENSC 283 Quiz #2

March 24, 2009

Name: ………………………………………. Student ID:……………………………….

Time: 45 minutes or less. Develop answers on available place. The quiz has 5% (bonus) of the total mark. Closed books & closed notes.

Problem 1 (40%):

The velocity field near a stagnation point may be written in the form:

$$u = \frac{U_0 x}{L} \qquad v = \frac{U_0 y}{L} \qquad U_0 \; and \; L \; are \; constants$$

a) Show that the acceleration vector is purely radial.
b) For the particular case *L=1.5 m*, if the acceleration at (*x,y*) = (1m, 1m) is 25 m/s^2, what is the value of U_0?

Acceleration:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

Solution:

(a) For two-dimensional steady flow, the acceleration components are

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial v} = \frac{U_0^2}{L^2}x$$

$$\frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial v} = \frac{U_0^2}{L^2}y$$

Therefore the resultant acceleration would be

$$\boldsymbol{a} = \frac{U_0^2}{L^2}(x\boldsymbol{i} + y\boldsymbol{j}) = \frac{U_0^2}{L^2}\boldsymbol{r}$$

which purely radial. Its magnitude would be

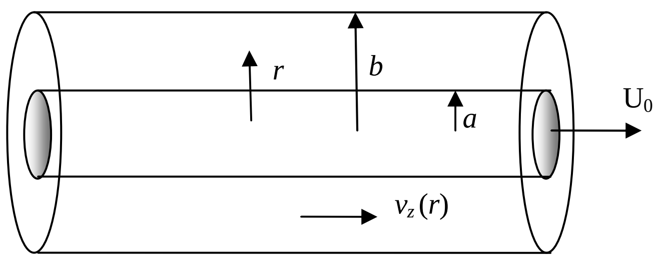
$$a = \frac{U_0^2}{L^2}\sqrt{x^2 + y^2}$$

(b) For a given resultant acceleration of 25 m/s^2, at $(x, y) = (1m, 1m)$, we obtain

$$25 = \frac{U_0^2}{1.5^2}\sqrt{1^2+1^2} \rightarrow U_0^2 = \frac{25 \times 1.5^2}{\sqrt{2}} \; or \; U_0 = 6.3\, m/s$$

Problem 2 (60%):

The viscous oil in figure below is set into steady motion by a concentric inner cylinder moving axially at velocity U_0 inside a fixed outer cylinder. Assuming constant pressure and density and a purely axial fluid motion, solve the axial momentum equation (given below) for the velocity distribution $v_z(r)$. What are the proper boundary conditions?



The continuity equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

The z-momentum equation in cylindrical coordinates:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Solution:

assumptions:

- Laminar flow
- Steady state
- Gravity in the z direction is negligible
- Constant pressure
- Constant properties
- Purely axial motion

Using these assumptions, $v_z(r)$ is only a function of r and momentum equation simplifies to

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = 0$$

subjected to the following boundary conditions

$$v_z(a) = U_0; \;\; v_z(0) = 0$$

The solution for the differential equation is

$$v_z = C_1 \, ln(r) + C_2$$

Applying the boundary conditions the final solution is

$$v_z = U_0 \frac{ln\left(\frac{r}{b}\right)}{ln\left(\frac{a}{b}\right)}$$