1.12 For low-speed (laminar) flow in a tube of radius r_o, the velocity u takes the form

$$u = B\frac{\Delta p}{\mu}\left(r_o^2 - r^2\right)$$

where μ is viscosity and Δp the pressure drop. What are the dimensions of B?

Solution: Using Table 1-2, write this equation in dimensional form:

$$\{u\} = \{B\}\frac{\{\Delta p\}}{\{\mu\}}\{r^2\}, \quad \text{or:} \quad \left\{\frac{L}{T}\right\} = \{B?\}\frac{\{M/LT^2\}}{\{M/LT\}}\{L^2\} = \{B?\}\left\{\frac{L^2}{T}\right\},$$

$$\text{or:} \quad \mathbf{\{B\} = \{L^{-1}\}} \quad \textit{Ans.}$$

The parameter B must have dimensions of inverse length. In fact, B is not a constant, it hides one of the variables in pipe flow. The proper form of the pipe flow relation is

$$u = C\frac{\Delta p}{L\mu}\left(r_o^2 - r^2\right)$$

where L is the *length of the pipe* and C is a dimensionless constant which has the theoretical laminar-flow value of (1/4)—see Sect. 6.4.

1.13 The efficiency η of a pump is defined as

$$\eta = \frac{Q\Delta p}{\text{Input Power}}$$

where Q is volume flow and Δp the pressure rise produced by the pump. What is η if $\Delta p = 35$ psi, Q = 40 L/s, and the input power is 16 horsepower?

Solution: The student should perhaps verify that $Q\Delta p$ has units of power, so that η is a dimensionless ratio. Then convert everything to consistent units, for example, BG:

$$Q = 40\ \frac{L}{s} = 1.41\ \frac{ft^2}{s}; \quad \Delta p = 35\ \frac{lbf}{in^2} = 5040\ \frac{lbf}{ft^2}; \quad \text{Power} = 16(550) = 8800\ \frac{ft\cdot lbf}{s}$$

$$\eta = \frac{(1.41\ ft^3/s)(5040\ lbf/ft^2)}{8800\ ft\cdot lbf/s} \approx 0.81 \quad \text{or} \quad \mathbf{81\%} \quad \textit{Ans.}$$

Similarly, one could convert to SI units: Q = 0.04 m^3/s, Δp = 241300 Pa, and input power = 16(745.7) = 11930 W, thus h = (0.04)(241300)/(11930) = **0.81**. *Ans.*