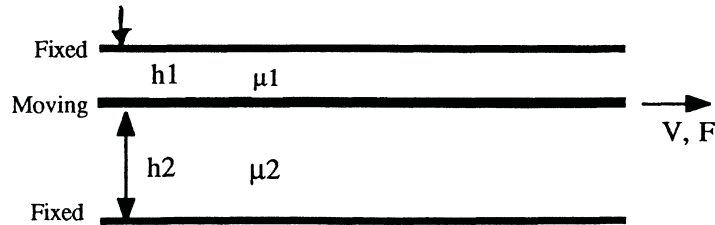**1.48**   A thin moving plate is separated from two fixed plates by two fluids of unequal viscosity and unequal spacing, as shown below. The contact area is A. Determine (a) the force required, and (b) is there a necessary relation between the two viscosity values?

Solution: (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left(\frac{\mu_1 V}{h_1} + \frac{\mu_2 V}{h_2} \right) A \quad Ans. \textbf{ (a)}$$

The formula is of course valid only for laminar (nonturbulent) steady viscous flow.
(b) Since the center plate separates the two fluids, they may have separate, unrelated shear stresses, and there is _no necessary relation_ between the two viscosities.

1.49   An amazing number of commercial and laboratory devices have been developed to measure fluid viscosity, as described in Ref. 27. Consider a concentric shaft, as in Prob. 1.47, but now fixed axially and rotated inside the sleeve. Let the inner and outer cylinders have radii r_i and r_o, respectively, with total sleeve length L. Let the rotational rate be Ω (rad/s) and the applied torque be M. Using these parameters, derive a theoretical relation for the viscosity μ of the fluid between the cylinders.

Solution: Assuming a linear velocity distribution in the annular clearance, the shear stress is

$$\tau = \mu \frac{\Delta V}{\Delta r} \approx \mu \frac{\Omega r_i}{r_o - r_i}$$

This stress causes a force $dF = \tau\, dA = \tau\, (r_i\, d\theta)L$ on each element of surface area of the inner shaft. The moment of this force about the shaft axis is $dM = r_i\, dF$. Put all this together:

$$M = \int r_i\, dF = \int_0^{2\pi} r_i \mu \frac{\Omega r_i}{r_o - r_i} r_i\, L\, d\theta = \frac{2\pi \mu \Omega r_i^3 L}{r_o - r_i}$$

Solve for the viscosity: $\mu \approx M(r_o - r_i)\big/\left\{2\pi\Omega r_i^3 L\right\} \quad Ans.$