P1.51 An approximation for the boundary-layer shape in Figs. 1.6*b* and P1.51 is the formula

$$u(y) \approx U\sin\left(\frac{\pi y}{2\delta}\right), \quad 0 \le y \le \delta$$

where U is the stream velocity far from the wall and δ is the boundary layer thickness, as in Fig. P.151. If the fluid is helium at 20°C and 1 atm, and if U = 10.8 m/s and δ = 3 mm, use the formula to (*a*) estimate the wall shear stress τ_w in Pa; and (*b*) find the position in the boundary layer where τ is one-half of τ_w.

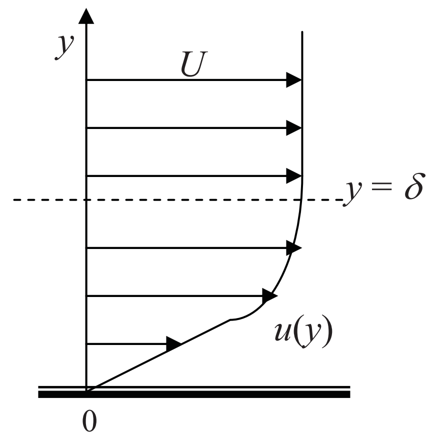

Fig. P1.51

Solution: From Table A.4, for helium, take R = 2077 $m^2/(s^2\text{-}K)$ and μ = 1.97E-5 kg/m-s.
(*a*) Then the wall shear stress is calculated as
A very small shear stress, but it has a profound effect on the flow pattern.

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0} = \mu\left(U\frac{\pi}{2\delta}\cos\frac{\pi y}{2\delta}\right)_{y=0} = \frac{\pi\,\mu U}{2\delta}$$

$$\text{Numerical values: } \tau_w = \frac{\pi(1.97E-5\,kg/m-s)(10.8\,m/s)}{2(0.003\,m)} = \mathbf{0.11\ Pa} \qquad Ans.(a)$$

(*b*) The variation of shear stress across the boundary layer is simply a cosine wave:

$$\tau(y) = \frac{\pi\mu U}{2\delta}\cos\left(\frac{\pi y}{2\delta}\right) = \tau_w\cos\left(\frac{\pi y}{2\delta}\right) = \frac{\tau_w}{2} \text{ when } \frac{\pi y}{2\delta} = \frac{\pi}{3}, \text{ or: } \mathbf{y} = \frac{\mathbf{2\delta}}{\mathbf{3}} \qquad Ans.(b)$$

1.52 The belt in Fig. P1.52 moves at steady velocity V and skims the top of a tank of oil of viscosity μ. Assuming a linear velocity profile, develop a simple formula for the belt-