Separating the variables, we may integrate:

$$\int_{\omega_o}^{w} \frac{d\omega}{\omega} = -\frac{\pi\mu r_o^4}{2hI_o \sin\theta}\int_0^t dt, \quad \text{or:} \quad \boldsymbol{\omega = \omega_o \exp\left[-\frac{5\pi\mu r_o^2 t}{3mh\sin\theta}\right]} \quad \textit{Ans.}$$

1.54* A disk of radius R rotates at angular velocity Ω inside an oil container of viscosity μ, as in Fig. P1.54. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

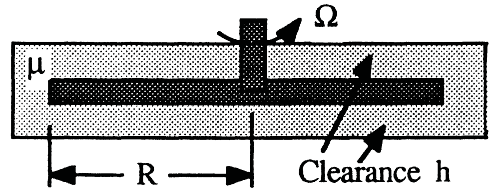

Fig. P1.54

Solution: At any r ≤ R, the viscous shear $\tau \approx \mu\Omega r/h$ on both sides of the disk. Thus,

$$d(\text{torque}) = dM = 2r\tau\, dA_w = 2r\frac{\mu\Omega r}{h}2\pi r\, dr,$$

$$\text{or:} \quad M = 4\pi\frac{\mu\Omega}{h}\int_0^R r^3\, dr = \boldsymbol{\frac{\pi\mu\Omega R^4}{h}} \quad \textit{Ans.}$$

P1.55 A block of weight *W* is being pulled over a table by another weight W_o, as shown in Fig. P1.55. Find an algebraic formula for the steady velocity *U* of the block if it slides on an oil film of thickness *h* and viscosity μ. The block bottom area *A* is in contact with the oil. Neglect the cord weight and the pulley friction.

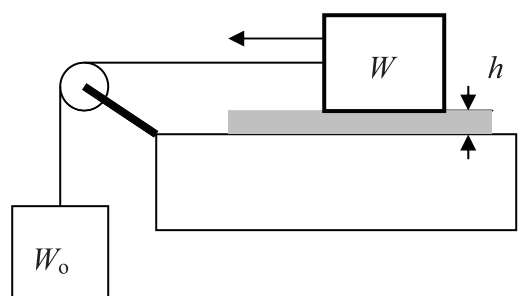

Fig. P1.55

Solution: This problem is a lot easier to *solve* than to set up and sketch. For steady motion, there is no acceleration, and the falling weight balances the viscous resistance of the oil film: