The complete (small-slope) solution to this problem is:

$$\boldsymbol{\eta = h\exp[-(\rho g/Y)^{1/2}x]}, \quad \text{where } h = (Y/\rho g)^{1/2}\cot\theta \quad \textit{Ans.}$$

The formula clearly satisfies the requirement that $\eta = 0$ if $x = \infty$. It requires "small slope" and therefore the contact angle should be in the range $70° < \theta < 110°$.

1.69 A solid cylindrical needle of diameter d, length L, and density ρ_n may "float" on a liquid surface. Neglect buoyancy and assume a contact angle of 0°. Calculate the maximum diameter needle able to float on the surface.

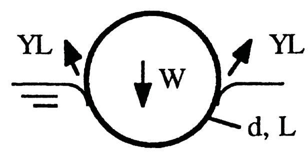

Fig. P1.69

Solution: The needle "dents" the surface downward and the surface tension forces are upward, as shown. If these tensions are nearly vertical, a vertical force balance gives:

$$\sum F_z = 0 = 2YL - \rho g\frac{\pi}{4}d^2L, \quad \text{or:} \quad \mathbf{d_{max}} \approx \sqrt{\frac{\mathbf{8Y}}{\boldsymbol{\pi\rho g}}} \quad \textit{Ans. (a)}$$

(b) Calculate d_{max} for a steel needle (SG ≈ 7.84) in water at 20°C. The formula becomes:

$$d_{max} = \sqrt{\frac{8Y}{\pi\rho g}} = \sqrt{\frac{8(0.073\text{ N/m})}{\pi(7.84 \times 998\text{ kg/m}^3)(9.81\text{ m/s}^2)}} \approx 0.00156\text{ m} \approx \mathbf{1.6\ mm} \quad \textit{Ans. (b)}$$

1.70 Derive an expression for the capillary-height change h, as shown, for a fluid of surface tension Y and contact angle θ between two parallel plates W apart. Evaluate h for water at 20°C if W = 0.5 mm.

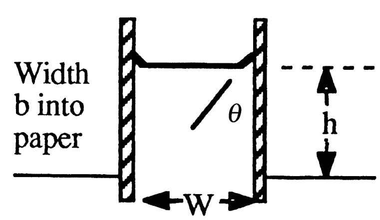

Fig. P1.70

Solution: With *b* the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$\rho gWhb = 2(Yb\cos\theta), \quad \text{or:} \quad h \approx \frac{\mathbf{2Y\cos\theta}}{\boldsymbol{\rho g W}} \quad \textit{Ans.}$$

For water at 20°C, Y ≈ 0.0728 N/m, $\rho g \approx 9790\ \text{N/m}^3$, and θ ≈ 0°. Thus, for W = 0.5 mm,

$$h = \frac{2(0.0728\ \text{N/m})\cos 0°}{(9790\ \text{N/m}^3)(0.0005\ \text{m})} \approx 0.030\ \text{m} \approx \mathbf{30\ mm} \quad \textit{Ans.}$$

1.71* A soap bubble of diameter D_1 coalesces with another bubble of diameter D_2 to form a single bubble D_3 with the same amount of air. For an isothermal process, express D_3 as a function of D_1, D_2, p_{atm}, and surface tension Y.

Solution: The masses remain the same for an isothermal process of an ideal gas:

$$m_1 + m_2 = \rho_1\upsilon_1 + \rho_2\upsilon_2 = m_3 = \rho_3\upsilon_3,$$

$$\text{or:}\quad \left(\frac{p_a + 4Y/r_1}{RT}\right)\left(\frac{\pi}{6}D_1^3\right) + \left(\frac{p_a + 4Y/r_2}{RT}\right)\left(\frac{\pi}{6}D_2^3\right) = \left(\frac{p_a + 4Y/r_3}{RT}\right)\left(\frac{\pi}{6}D_3^3\right)$$

The temperature cancels out, and we may clean up and rearrange as follows:

$$\mathbf{p_a D_3^3 + 8YD_3^2 = \left(p_a D_2^3 + 8YD_2^2\right) + \left(p_a D_1^3 + 8YD_1^2\right)} \quad \textit{Ans.}$$

This is a cubic polynomial with a known right hand side, to be solved for D_3.

1.72 Early mountaineers boiled water to estimate their altitude. If they reach the top and find that water boils at 84°C, approximately how high is the mountain?

Solution: From Table A-5 at 84°C, vapor pressure $p_v \approx 55.4$ kPa. We may use this value to interpolate in the standard altitude, Table A-6, to estimate

$$z \approx \mathbf{4800\ m} \quad \textit{Ans.}$$

1.73 A small submersible moves at velocity V in 20°C water at 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is Ca ≈ 0.25. At what velocity will cavitation bubbles form? Will the body cavitate if V = 30 m/s and the water is cold (5°C)?

Solution: From Table A-5 at 20°C read $p_v = 2.337$ kPa. By definition,

$$Ca_{crit} = 0.25 = \frac{2(p_a - p_v)}{\rho V^2} = \frac{2(131000 - 2337)}{(998\ \text{kg/m}^3)V^2}, \quad \text{solve } V_{crit} \approx \mathbf{32.1\ m/s} \quad \textit{Ans.}\ \text{(a)}$$