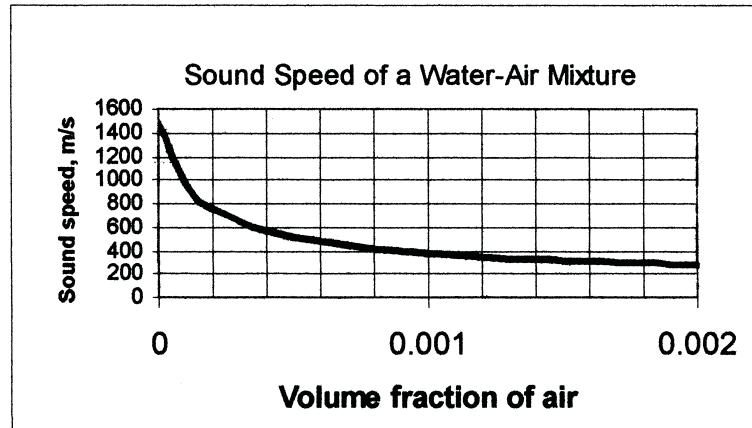


(b) For the given data, a plot of sound speed versus gas volume fraction is as follows:



The difference in air and water compressibility is so great that the speed drop-off is quite sharp.

**1.80\*** A two-dimensional steady velocity field is given by  $u = x^2 - y^2$ ,  $v = -2xy$ . Find the streamline pattern and sketch a few lines. [*Hint*: The differential equation is exact.]

**Solution:** Equation (1.44) leads to the differential equation:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dx}{x^2 - y^2} = \frac{dy}{-2xy}, \quad \text{or: } (2xy)dx + (x^2 - y^2)dy = 0$$

As hinted, this equation is *exact*, that is, it has the form  $dF = (\partial F/\partial x)dx + (\partial F/\partial y)dy = 0$ . We may check this readily by noting that  $\partial/\partial y(2xy) = \partial/\partial x(x^2 - y^2) = 2x = \partial^2 F/\partial x \partial y$ . Thus we may integrate to give the formula for streamlines:

$$F = x^2y - y^3/3 + \text{constant} \quad \text{Ans.}$$

This represents (inviscid) flow in a series of  $60^\circ$  corners, as shown in Fig. E4.7a of the text. [This flow is also discussed at length in Section 4.7.]

**1.81** Repeat Ex. 1.13 by letting the velocity components increase linearly with time:

$$\mathbf{V} = Kxt\mathbf{i} - Kyt\mathbf{j} + 0\mathbf{k}$$

**Solution:** The flow is unsteady and two-dimensional, and Eq. (1.44) still holds:

$$\text{Streamline: } \frac{dx}{u} = \frac{dy}{v}, \quad \text{or: } \frac{dx}{Kxt} = \frac{dy}{-Kyt}$$