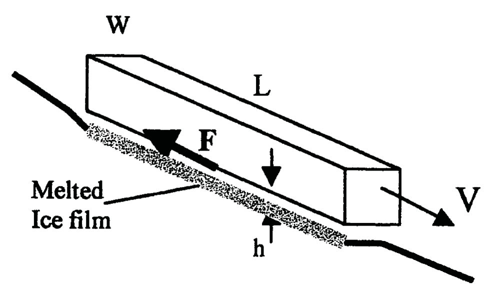
C1.2 When a person ice-skates, the ice surface actually melts beneath the blades, so that he or she skates on a thin film of water between the blade and the ice. (a) Find an expression for total friction force F on the bottom of the blade as a function of skater velocity V, blade length L, water film thickness h, water viscosity μ, and blade width W. (b) Suppose a skater of mass *m*, moving at constant speed V_o, suddenly stands stiffly with skates pointed directly forward and allows herself to coast to a stop. Neglecting air resistance, how far will she travel (on *two* blades) before she stops? Give the answer X as a function of (V_o, *m*, L, h, μ, W). (c) Compute X for the case V_o = 4 m/s, m = 100 kg, L = 30 cm, W = 5 mm, and h = 0.1 mm. Do you think our assumption of negligible air resistance was a good one?

Solution: (a) The skate bottom and the melted ice are like two parallel plates:

$$\tau = \mu\frac{V}{h}, \quad F = \tau A = \frac{\mu VLW}{h} \quad \textit{Ans.}\text{ (a)}$$

(b) Use **F** = m**a** to find the stopping distance:



$$\Sigma F_x = -F = -\frac{2\mu VLW}{h} = ma_x = m\frac{dV}{dt}$$

(*the* '2' *is for two blades*)

Separate and integrate once to find the velocity, once again to find the distance traveled:

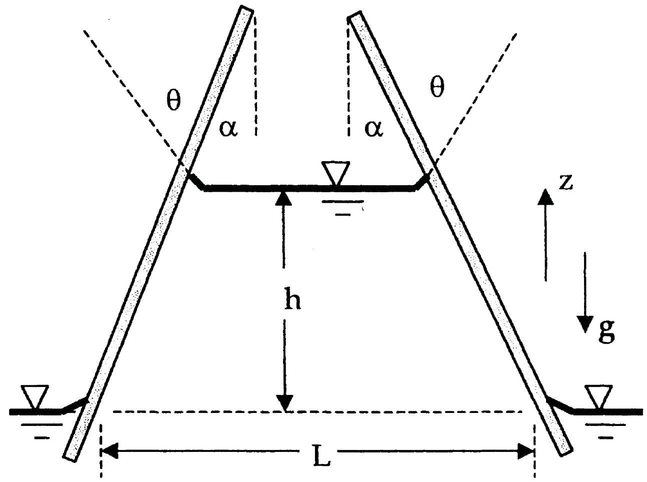
$$\int \frac{dV}{V} = -\int \frac{2\mu LW}{mh}dt, \quad or: \quad V = V_o e^{\frac{-2\mu LW}{mh}t}, \quad X = \int_0^\infty V\,dt = \frac{V_o mh}{2\mu LW} \quad \textit{Ans.}\text{ (b)}$$

(c) Apply our specific numerical values to a 100-kg (!) person:

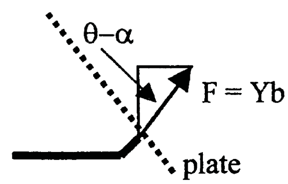
$$X = \frac{(4.0\ m/s)(100\ kg)(0.0001\ m)}{2(1.788E\text{–}3\ kg/m\cdot s)(0.3\ m)(0.005\ m)} = \mathbf{7460\ m\ (!)} \quad \textit{Ans.}\text{ (c)}$$

We could coast to the next town on ice skates! It appears that our assumption of negligible air drag was grossly incorrect.

C1.3 Two thin flat plates are tilted at an angle α and placed in a tank of known surface tension Y and contact angle θ, as shown. At the free surface of the liquid in the tank, the two plates are a distance *L* apart, and of width *b* into the paper. (a) What is the total *z*-directed force, due to surface tension, acting on the liquid column between plates? (b) If the liquid density is ρ, find an expression for Y in terms of the other variables.



Solution: (a) Considering the right side of the liquid column, the surface tension acts tangent to the local surface, that is, along the dashed line at right. This force has magnitude $F = Yb$, as shown. Its vertical component is $F\cos(\theta - \alpha)$, as shown. There are two plates. Therefore, the total z-directed force on the liquid column is



$$\mathbf{F_{vertical} = 2Yb\cos(\theta - \alpha)} \quad \textit{Ans.} \text{ (a)}$$

(b) The vertical force in (a) above holds up the entire weight of the liquid column between plates, which is $W = \rho g\{bh(L - h\tan\alpha)\}$. Set W equal to F and solve for

$$\mathbf{U = [\rho g b h(L - h\tan\alpha)]/[2\cos(\theta - \alpha)]} \quad \textit{Ans.} \text{ (b)}$$

C1.4 Oil of viscosity μ and density ρ drains steadily down the side of a tall, wide vertical plate, as shown. The film is fully developed, that is, its thickness δ and velocity profile w(x) are independent of distance z down the plate. Assume that the atmosphere offers no shear resistance to the film surface.

(a) Sketch the approximate shape of the velocity profile w(x), keeping in mind the boundary conditions.