2.120 A uniform wooden beam (SG = 0.65) is 10 cm by 10 cm by 3 m and hinged at A. At what angle will the beam float in 20°C water?

Solution: The total beam volume is $3(.1)^2 = 0.03\ m^3$, and therefore its weight is W = (0.65)(9790)(0.03) = 190.9 N, acting at the centroid, 1.5 m down from point A. Meanwhile, if the submerged length is H, the buoyancy is $B = (9790)(0.1)^2H = 97.9H$ newtons, acting at H/2 from the lower end. Sum moments about point A:

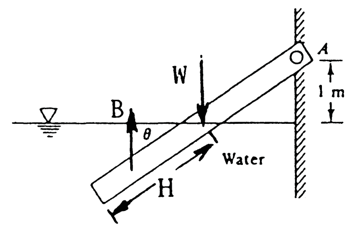

Fig. P2.120

$$\sum M_A = 0 = (97.9H)(3.0 - H/2)\cos\theta - 190.9(1.5\cos\theta),$$

$$\text{or:}\quad H(3 - H/2) = 2.925, \quad \text{solve for} \quad H \approx 1.225\ m$$

Geometry: 3 − H = 1.775 m is out of the water, or: $\sin\theta = 1.0/1.775$, or $\theta \approx$ **34.3°** *Ans.*

2.121 The uniform beam in the figure is of size L by h by b, with $b,h \ll L$. A uniform heavy sphere tied to the left corner causes the beam to float exactly on its diagonal. Show that this condition requires (a) $\gamma_b = \gamma/3$; and (b) $D = [Lhb/\{\pi(SG - 1)\}]^{1/3}$.

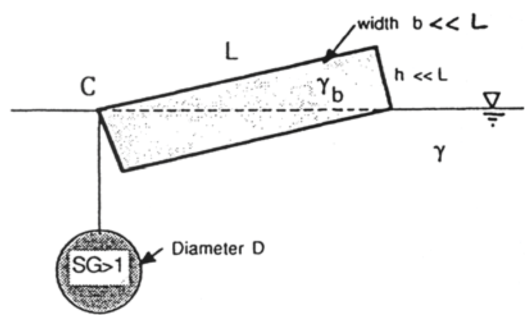

Fig. P2.121

Solution: The beam weight $W = \gamma_b Lhb$ and acts in the center, at L/2 from the left corner, while the buoyancy, being a perfect triangle of displaced water, equals $B = \gamma Lhb/2$ and acts at L/3 from the left corner. Sum moments about the left corner, point C:

$$\sum M_C = 0 = (\gamma_b Lhb)(L/2) - (\gamma Lhb/2)(L/3), \quad \text{or:} \quad \boldsymbol{\gamma_b = \gamma/3} \quad \textit{Ans. (a)}$$

Then summing vertical forces gives the required string tension T on the left corner:

$$\sum F_z = 0 = \gamma Lbh/2 - \gamma_b Lbh - T, \quad \text{or} \quad T = \gamma Lbh/6 \quad \text{since} \quad \gamma_b = \gamma/3$$

$$\text{But also} \quad T = (W - B)_{sphere} = (SG - 1)\gamma\frac{\pi}{6}D^3, \quad \text{so that} \quad D = \left[\frac{\mathbf{Lhb}}{\boldsymbol{\pi}(\mathbf{SG - 1})}\right]^{1/3} \quad \textit{Ans. (b)}$$
