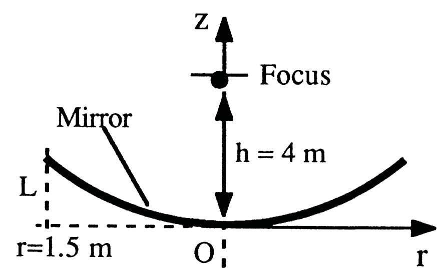
2.158* It is desired to make a 3-m-diameter parabolic telescope mirror by rotating molten glass in rigid-body motion until the desired shape is achieved and then cooling the glass to a solid. The focus of the mirror is to be 4 m from the mirror, measured along the centerline. What is the proper mirror rotation rate, in rev/min?



Solution: We have to review our math book, or Mark's Manual, to recall that the *focus* F of a parabola is the point for which all points on the parabola are equidistant from both the focus and a so-called "directrix" line (which is one focal length below the mirror).
For the focal length *h* and the *z-r* axes shown in the figure, the equation of the parabola is given by $\mathbf{r^2 = 4hz}$, with $h = 4$ m for our example.
Meanwhile the equation of the free-surface of the liquid is given by $\mathbf{z = r^2\Omega^2/(2g)}$.
Set these two equal to find the proper rotation rate:

$$z = \frac{r^2\Omega^2}{2g} = \frac{r^2}{4h}, \quad \text{or:} \quad \Omega^2 = \frac{g}{2h} = \frac{9.81}{2(4)} = 1.226$$

$$Thus \quad \Omega = 1.107\ \frac{\text{rad}}{\text{s}}\left(\frac{60}{2\pi}\right) = \mathbf{10.6\ rev/min} \quad Ans.$$

The focal point F is far above the mirror itself. If we put in $r = 1.5$ m and calculate the mirror depth "L" shown in the figure, we get L ≈ 14 centimeters.

2.159 The three-legged manometer in Fig. P2.159 is filled with water to a depth of 20 cm. All tubes are long and have equal small diameters. If the system spins at angular velocity Ω about the central tube, (a) derive a formula to find the change of height in the tubes; (b) find the height in cm in each tube if Ω = 120 rev/min. [*HINT*: The central tube must supply water to *both* the outer legs.]

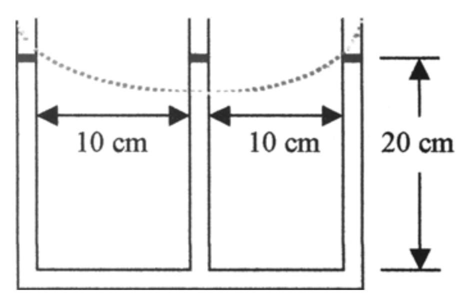


Fig. P2.159

Solution: (a) The free-surface during rotation is visualized as the **red** dashed line in Fig. P2.159. The outer right and left legs experience an increase which is one-half that of the central leg, or $\Delta h_O = \Delta h_C/2$. The total displacement between outer and center menisci is, from Eq. (2.64) and Fig. 2.23, equal to $\Omega^2R^2/(2g)$. The center meniscus falls two-thirds of this amount and feeds the outer tubes, which each rise one-third of this amount above the rest position:

$$\boldsymbol{\Delta h_{outer} = \frac{1}{3}\Delta h_{total} = \frac{\Omega^2 R^2}{6g} \qquad \Delta h_{center} = -\frac{2}{3}\Delta h_{total} = -\frac{\Omega^2 R^2}{3g}} \quad \textit{Ans. (a)}$$

For the particular case $R = 10$ cm and $\Omega = 120$ r/min $= (120)(2\pi/60) = 12.57$ rad/s, we obtain

$$\frac{\Omega^2 R^2}{2g} = \frac{(12.57\text{ rad/s})^2(0.1\text{ m})^2}{2(9.81\text{ m/s}^2)} = 0.0805\text{ m};$$

$$\Delta h_O \approx \mathbf{0.027\ m}\ (up) \quad \Delta h_C \approx \mathbf{-0.054\ m}\ (down) \quad \textit{Ans. (b)}$$
