Solution: (a) The resultant force F, may be found by simply applying the hydrostatic relation

$$F = \gamma h_{CG} A = (9790\ \text{N/m}^3)(3 + 1.5\ \text{m})(5\ \text{m} \times 2\ \text{m}) = 440{,}550\ \text{N} = \mathbf{441\ kN} \quad \textit{Ans.}\ \text{(a)}$$

(b) The horizontal force acts as though BC were vertical, thus h_{CG} is halfway down from C and acts on the projected area of BC.

$$F_H = (9790)(4.5)(3 \times 2) = 264{,}330\ \text{N} = \mathbf{264\ kN} \quad \textit{Ans.}\ \text{(b)}$$

The vertical force is equal to the weight of fluid above BC,

$$F_V = (9790)[(3)(4) + (1/2)(4)(3)](2) = 352{,}440 = \mathbf{352\ kN} \quad \textit{Ans.}\ \text{(b)}$$

The resultant is the same as part (a): $F = [(264)^2 + (352)^2]^{1/2} = \mathbf{441\ kN.}$

2.58 In Fig. P2.58, weightless cover gate AB closes a circular opening 80 cm in diameter when weighed down by the 200-kg mass shown. What water level *h* will dislodge the gate?

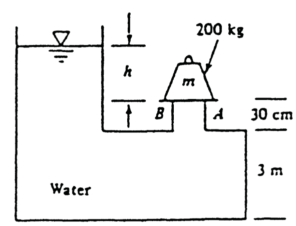


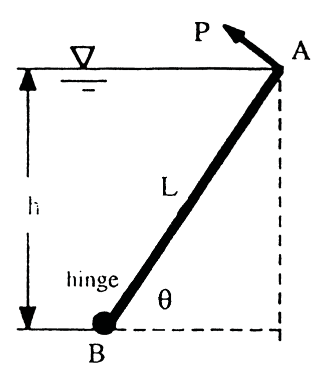
Fig. P2.58

Solution: The centroidal depth is exactly equal to *h* and force F will be upward on the gate. Dislodging occurs when F equals the weight:

$$F = \gamma h_{CG} A_{gate} = (9790\ \text{N/m}^3)\ h \frac{\pi}{4}(0.8\ \text{m})^2 = W = (200)(9.81)\ \text{N}$$

$$\text{Solve for} \quad h = \mathbf{0.40\ m} \quad \textit{Ans.}$$

2.59 Gate AB has length L, width b into the paper, is hinged at B, and has negligible weight. The liquid level h remains at the top of the gate for any angle θ. Find an analytic expression for the force P, perpendicular to AB, required to keep the gate in equilibrium.



Solution: The centroid of the gate remains at distance L/2 from A and depth h/2 below the surface. For any θ, then, the hydrostatic force is $F = \gamma(h/2)Lb$. The moment of inertia of the gate is $(1/12)bL^3$, hence $y_{CP} = -(1/12)bL^3\sin\theta/[(h/2)Lb]$, and the center of pressure is $(L/2 - y_{CP})$ from point B. Summing moments about hinge B yields

$$PL = F(L/2 - y_{CP}), \quad \text{or:} \quad \mathbf{P = (\gamma hb/4)(L - L^2\sin\theta/3h)} \quad \textit{Ans.}$$

P2.60 In 1960, Auguste and Jacques Picard's self-propelled bathyscaphe *Trieste* set a record by descending to a depth of 35,800 feet in the Pacific Ocean, near Guam. The passenger sphere was 7 ft in diameter, 6 inches thick, and had a window 16 inches in diameter. (*a*) Estimate the hydrostatic force on the window at that depth. (*b*) If the window is vertical, how far below its center is the center of pressure?

Solution: At the surface, the density of seawater is about 1025 kg/m^3 (1.99 slug/ft^3). Atmospheric pressure is about 2116 lbf/ft^2. We could use these values, or estimate from Eq. (1.19) that the density at depth would be about 4.6% more, or 2.08 slug/ft^3. We could average these two to 2.035 slug/ft^3. The pressure at that depth would thus be approximately

$$p = p_a + \rho_{avg}\, g\, h = 2116 + (2.035\frac{slug}{ft^3})(32.2\frac{ft}{s^2})(35800\ ft) \approx 2{,}350{,}000\ \frac{lbf}{ft^2}$$

(*a*) This pressure, times the area of the 16-inch window, gives the desired force.

$$F_{window} = p_{CG}\, A = (2350000\frac{lbf}{ft^2})[\frac{\pi}{4}(\frac{16}{12}ft)^2] = \mathbf{3{,}280{,}000\ \ lbf} \quad \textit{Ans.}(a)$$

Quite a lot of force, but the bathyscaphe was well designed.

(*b*) The distance down to the center of pressure on the window follows from Eq. (2.27):

$$y_{CP} = -\gamma\sin\theta\frac{I_{xx}}{F} = -[2.035*32.2\frac{lbf}{ft^3}]\sin(90^\circ)\frac{(\pi/4)(8/12\ ft)^4}{3280000\, lbf} = \mathbf{-3.2E-6\ \ ft.} \quad \textit{Ans.}(b)$$

The center of pressure at this depth is only 38 micro inches below the center of the window.
