2.84 Determine (a) the total hydrostatic force on curved surface AB in Fig. P2.84 and (b) its line of action. Neglect atmospheric pressure and assume unit width into the paper.

Solution: The horizontal force is


Fig. P2.84

$$
\mathrm{F}_{\mathrm{H}}=\gamma \mathrm{h}_{\mathrm{CG}} \mathrm{~A}_{\text {vert }}=\left(9790 \mathrm{~N} / \mathrm{m}^{3}\right)(0.5 \mathrm{~m})\left(1 \times 1 \mathrm{~m}^{2}\right)=4895 \mathrm{~N} \text { at } 0.667 \mathrm{~m} \text { below } \mathrm{B} .
$$

For the cubic-shaped surface $A B$, the weight of water above is computed by integration:

$$
\begin{aligned}
\mathrm{F}_{\mathrm{V}} & =\gamma \mathrm{b} \int_{0}^{1}\left(1-\mathrm{x}^{3}\right) \mathrm{dx}=\frac{3}{4} \gamma \mathrm{~b} \\
& =(3 / 4)(9790)(1.0)=7343 \mathrm{~N}
\end{aligned}
$$



The line of action (water centroid) of the vertical force also has to be found by integration:

$$
\bar{x}=\frac{\int x d A}{\int \mathrm{dA}}=\frac{\int_{0}^{1} x\left(1-x^{3}\right) d x}{\int_{0}^{1}\left(1-x^{3}\right) d x}=\frac{3 / 10}{3 / 4}=0.4 \mathrm{~m}
$$

The vertical force of 7343 N thus acts at 0.4 m to the right of point A , or 0.6 m to the left of B, as shown in the sketch above. The resultant hydrostatic force then is

$$
\mathrm{F}_{\text {total }}=\left[(4895)^{2}+(7343)^{2}\right]^{1 / 2}=\mathbf{8 8 2 5} \mathbf{N} \text { acting at } \mathbf{5 6 . 3 1 ^ { \circ }} \text { down and to the right. Ans. }
$$

This result is shown in the sketch at above right. The line of action of F strikes the vertical above point A at 0.933 m above A, or 0.067 m below the water surface.
2.85 Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle panel at the bottom of the water tank in Fig. P2.85.

Solution: The horizontal component is

$$
\begin{aligned}
\mathrm{F}_{\mathrm{H}} & =\gamma \mathrm{h}_{\mathrm{CG}} \mathrm{~A}_{\text {vert }}=(9790)(6)(2 \times 6) \\
& =705000 \mathbf{N} \quad \text { Ans. }(\mathrm{a})
\end{aligned}
$$



Fig. P2.85

