3.155 Bernoulli's 1738 treatise *Hydrodynamica* contains many excellent sketches of flow patterns. One, however, redrawn here as Fig. P3.155, seems physically misleading. What is wrong with the drawing?

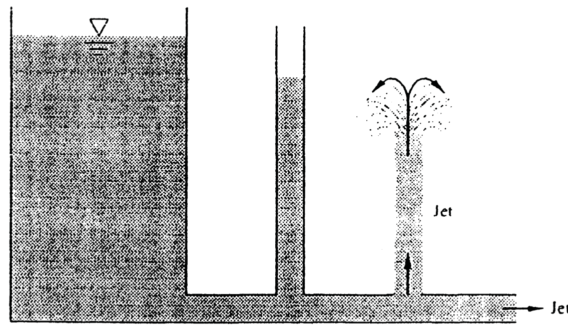

Fig. P3.155

Solution: If friction is neglected and the exit pipe is fully open, then pressure in the closed "piezometer" tube would be atmospheric and the fluid would not rise at all in the tube. The open jet coming from the hole in the tube would have V ≈ √(2gh) and would rise up to nearly the same height as the water in the tank.

P3.156 Extend Prob. 3.13 as follows. (*a*) Use Bernoulli's equation to estimate the elevation of the water surface above the exit of the bottom cone. (*b*) Then estimate the time required for the water surface to drop 20 cm in the cylindrical tank. If you fail to solve part (*a*), assume that the initial elevation above the exit is 52 cm. Neglect the possible contraction and nonuniformity of the exit jet mentioned in Ex. 3.21.

Solution: This is a "Torricelli" flow, like Ex. 3.21. Using the continuity relation in Prob. 3.tank, we found that $V \approx 3.2$ m/s. (*a*) Thus we know everything except $\Delta z = h_{\text{cyl}} + h_{\text{cone}}$:

$$V = \sqrt{2g\,\Delta z} \approx 3.2\,m/s = \sqrt{2(9.81\,m/s^2)\Delta z}\;,\text{ solve for }\Delta z = 0.52\,m = \mathbf{52\,cm} \quad Ans.(a)$$

(*b*) The time Δt to drop from 52 cm to 32 cm could be obtained by numerical quadrature, or, better, we could solve for Δt analytically:

$$Q_{out} = \frac{\pi}{4}D_{exit}^2 V = -\frac{d}{dt}\left(\frac{\pi}{4}D^2 h\right) = \frac{\pi}{4}D_{exit}^2\sqrt{2g\Delta z} = -\frac{\pi}{4}D^2\frac{dh}{dt} = \frac{\pi}{4}D^2\frac{d(\Delta z)}{dt}$$

$$\text{Separate \& integrate: } \int_{0.52}^{0.32}\frac{d(\Delta z)}{\sqrt{\Delta z}} = -\sqrt{2g}\left(\frac{D_{exit}}{D}\right)^2\int_0^{\Delta t} dt$$

$$\text{Result: } \Delta t = \frac{2(\sqrt{0.52}-\sqrt{0.32})}{\sqrt{2g}}\left(\frac{D}{D_{exit}}\right)^2 = \frac{0.3108}{\sqrt{2(9.81)}}\left(\frac{0.2cm}{0.03cm}\right)^2 = \Delta t = \mathbf{3.12\,s} \quad Ans.(b)$$
