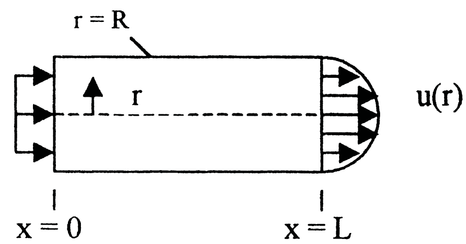
3.15 Water flows steadily through the round pipe in the figure. The entrance velocity is V_o. The exit velocity approximates turbulent flow, $u = u_{max}(1 - r/R)^{1/7}$. Determine the ratio U_o/u_{max} for this incompressible flow.



Solution: Inlet and outlet flow must balance:

$$Q_1 = Q_2, \quad or: \quad \int_0^R U_o 2\pi r\, dr = \int_0^R u_{max}\left(1-\frac{r}{R}\right)^{1/7} 2\pi r\, dr, \quad or: \quad U_o \pi R^2 = u_{max}\frac{49\pi}{60}R^2$$

Cancel and rearrange for this assumed incompressible pipe flow:

$$\frac{U_o}{u_{\max}} = \mathbf{\frac{49}{60}} \quad Ans.$$

3.16 An incompressible fluid flows past an impermeable flat plate, as in Fig. P3.16, with a uniform inlet profile $u = U_o$ and a cubic polynomial exit profile

$$u \approx U_o\left(\frac{3\eta - \eta^3}{2}\right) \quad \text{where } \eta = \frac{y}{\delta}$$

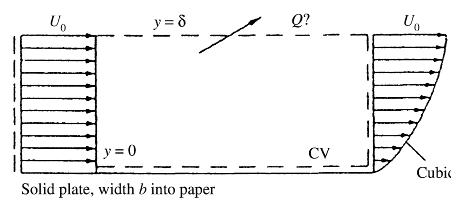


Fig. P3.16

Compute the volume flow Q across the top surface of the control volume.

Solution: For the given control volume and incompressible flow, we obtain

$$0 = Q_{top} + Q_{right} - Q_{left} = Q + \int_0^\delta U_o\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right) b\, dy - \int_0^\delta U_o b\, dy$$

$$= Q + \frac{5}{8}U_o b\delta - U_o b\delta, \quad \text{solve for } \mathbf{Q = \frac{3}{8}U_o b\delta} \quad Ans.$$

3.17 Incompressible steady flow in the inlet between parallel plates in Fig. P3.17 is uniform, $u = U_o = 8$ cm/s, while downstream the flow develops into the parabolic laminar profile $u = az(z_o - z)$, where a is a constant. If $z_o = 4$ cm and the fluid is SAE 30 oil at 20°C, what is the value of u_{max} in cm/s?

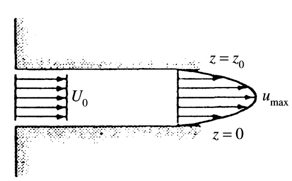


Fig. P3.17