3.160 The air-cushion vehicle in Fig. P3.160 brings in sea-level standard air through a fan and discharges it at high velocity through an annular skirt of $3-\mathrm{cm}$ clearance. If the vehicle weighs 50 kN , estimate (a) the required airflow rate and (b) the fan power in kW .

Solution: The air inside at section 1 is nearly stagnant ( $\mathrm{V} \approx 0$ ) and supports the


Fig. P3. 160 weight and also drives the flow out of the interior into the atmosphere:
$\mathrm{p}_{1} \approx \mathrm{p}_{\mathrm{o} 1}: \quad \mathrm{p}_{\mathrm{o} 1}-\mathrm{p}_{\mathrm{atm}}=\frac{\text { weight }}{\text { area }}=\frac{50,000 \mathrm{~N}}{\pi(3 \mathrm{~m})^{2}}=\frac{1}{2} \rho \mathrm{~V}_{\text {exit }}^{2}=\frac{1}{2}(1.205) \mathrm{V}_{\text {exit }}^{2} \approx 1768 \mathrm{~Pa}$
Solve for $\quad \mathrm{V}_{\text {exit }} \approx 54.2 \mathrm{~m} / \mathrm{s}$, whence $\mathrm{Q}_{\mathrm{e}}=\mathrm{A}_{\mathrm{e}} \mathrm{V}_{\mathrm{e}}=\pi(6)(0.03)(54.2)=30.6 \frac{\mathrm{~m}^{3}}{\mathrm{~s}}$
Then the power required by the fan is $\mathrm{P}=\mathrm{Qe} \Delta \mathrm{p}=(30.6)(1768) \approx \mathbf{5 4 0 0 0} \mathbf{~ W}$ Ans.
3.161 A necked-down section in a pipe flow, called a venturi, develops a low throat pressure which can aspirate fluid upward from a reservoir, as in Fig. P3.161. Using Bernoulli's equation with no losses, derive an expression for the velocity $V_{1}$ which is just sufficient to bring reservoir fluid into the throat.


Fig. P3. 161

Solution: Water will begin to aspirate into the throat when $\mathrm{pa}-\mathrm{p} 1=\rho \mathrm{gh}$. Hence:
Volume flow: $\quad \mathrm{V}_{1}=\mathrm{V}_{2}\left(\mathrm{D}_{2} / \mathrm{D}_{1}\right)^{2} ; \quad$ Bernoulli $(\Delta \mathrm{z}=0): \quad \mathrm{p}_{1}+\frac{1}{2} \rho \mathrm{~V}_{1}^{2} \approx \mathrm{p}_{\mathrm{atm}}+\frac{1}{2} \rho \mathrm{~V}_{2}^{2}$

Solve for $\mathrm{p}_{\mathrm{a}}-\mathrm{p}_{1}=\frac{\rho}{2}\left(\alpha^{4}-1\right) \mathrm{V}_{2}^{2} \geq \rho \mathrm{gh}, \quad \alpha=\frac{\mathrm{D}_{2}}{\mathrm{D}_{1}}, \quad$ or: $\quad \mathrm{V}_{\mathbf{2}} \geq \sqrt{\frac{\mathbf{2 g h}}{\alpha^{4}-\mathbf{1}}}$ Ans.

$$
\text { Similarly, } \quad \mathbf{V}_{1, \min }=\alpha^{2} V_{2, \min }=\sqrt{\frac{2 \mathbf{g h}}{1-\left(\mathbf{D}_{1} / \mathbf{D}_{2}\right)^{4}}} \text { Ans. }
$$

3.162 Suppose you are designing a $3 \times 6$ - ft air-hockey table, with $1 / 16$-inch-diameter holes spaced every inch in a rectangular pattern (2592 holes total), the required jet speed from each hole is $50 \mathrm{ft} / \mathrm{s}$. You must select an appropriate blower. Estimate the volumetric flow rate (in $\mathrm{ft}^{3} / \mathrm{min}$ ) and
 pressure rise (in psi) required. Hint: Assume the air is stagnant in the large manifold under the table surface, and neglect frictional losses.

Solution: Assume an air density of about sea-level, $\mathbf{0 . 0 0 2 3 5} \mathbf{s l u g} / \mathbf{f t}^{\mathbf{3}}$. Apply Bernoulli's equation through any single hole, as in the figure:

$$
\begin{gathered}
p_{1}+\frac{\rho}{f} V_{1}^{2}=p_{a}+\frac{\rho}{2} V_{\text {jet }}^{2}, \quad \text { or: } \\
\Delta p_{\text {required }}=p_{1}-p_{a}=\frac{\rho}{2} V_{\text {jet }}^{2}=\frac{0.00235}{2}(50)^{2}=2.94 \frac{\mathrm{lbf}}{f t^{2}}=\mathbf{0 . 0 2 0 4} \frac{\mathbf{l b f}}{\mathbf{i n}^{2}} \text { Ans. }
\end{gathered}
$$

The total volume flow required is

$$
\begin{aligned}
Q=V A_{1-\text { hole }}(\# \text { of holes }) & =\left(50 \frac{f t}{s}\right) \frac{\pi}{4}\left(\frac{1 / 16}{12} f t\right)^{2}(2592 \text { holes }) \\
& =2.76 \frac{f t^{3}}{s}=\mathbf{1 6 6} \frac{\mathbf{f t}^{3}}{\mathbf{m i n}} \text { Ans. }
\end{aligned}
$$

It wasn't asked, but the power required would be $P=\mathrm{Q} \Delta \mathrm{p}=\left(2.76 \mathrm{ft}^{3} / \mathrm{s}\right)\left(2.94 \mathrm{lbf} / \mathrm{ft}^{2}\right)=$ $8.1 \mathrm{ft} \cdot \mathrm{lbf} / \mathrm{s}$, or about 11 watts.

