The integral term $\int \frac{\partial V}{\partial t}\, ds \approx \frac{dV_1}{dt}\, h$ is very small and will be neglected, and $p_1 = p_2$.. Then

$V_1 \approx \left[\frac{2gh}{\alpha - 1}\right]^{1/2}$, where $\alpha = (D/d)^4$; but also $V_1 = -\frac{dh}{dt}$, separate and integrate:

$$\int_{h_o}^{h} \frac{dh}{h^{1/2}} = -\left[\frac{2g}{\alpha-1}\right]^{1/2} \int_{o}^{t} dt, \quad \text{or:} \quad \mathbf{h = \left[h_o^{1/2} - \left\{\frac{g}{2(\alpha-1)}\right\}^{1/2} t\right]^2}, \quad \alpha = \left(\frac{D}{d}\right)^4 \quad \textit{Ans.} \text{ (a)}$$

(b) the tank is empty when [] = 0 in (a) above, or $\mathbf{t_{final} = [2(\alpha - 1)g/h_o]^{1/2}}$. *Ans*. (b)

3.180 The large tank of incompressible liquid in Fig. P3.180 is at rest when, at $t = 0$, the valve is opened to the atmosphere. Assuming $h \approx$ constant (negligible velocities and accelerations in the tank), use the unsteady frictionless Bernoulli equation to derive and solve a differential equation for $V(t)$ in the pipe.

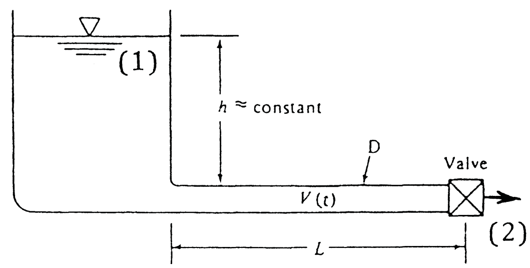

Fig. P3.180

Solution: Write unsteady Bernoulli from 1 to 2:

$$\int_1^2 \frac{\partial V}{\partial t}\, ds + \frac{V_2^2}{2} + gz_2 \approx \frac{V_1^2}{2} + gz_1, \quad \text{where } p_1 = p_2,\ V_1 \approx 0,\ z_2 \approx 0, \text{ and } z_1 = h = \text{const}$$

The integral approximately equals $\frac{dV}{dt}L$, so the diff. eqn. is $2L\frac{dV}{dt} + V^2 = 2gh$

This first-order ordinary differential equation has an exact solution for V = 0 at t = 0:

$$\mathbf{V = V_{final} \tanh\left(\frac{V_{final}\, t}{2L}\right)}, \quad \text{where } \mathbf{V_{final} = \sqrt{2gh}} \quad \textit{Ans.}$$

3.181 Modify Prob. 3.180 as follows. Let the top of the tank be enclosed and under constant gage pressure p_o. Repeat the analysis to find $V(t)$ in the pipe.