

Solution: (a) For incompressible flow, the volume flow is the same at piston and exit:

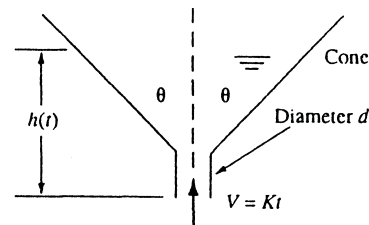
$$Q = 6 \frac{\text{cm}^3}{\text{s}} = 0.366 \frac{\text{in}^3}{\text{s}} = A_1 V_1 = \frac{\pi}{4} (0.75 \text{ in})^2 V_1, \quad \text{solve } V_{\text{piston}} = \mathbf{0.83 \frac{\text{in}}{\text{s}}} \quad \text{Ans. (a)}$$

(b) If there is 10% leakage, the piston must deliver both needle flow and leakage:

$$A_1 V_1 = Q_{\text{needle}} + Q_{\text{clearance}} = 6 + 0.1(6) = 6.6 \frac{\text{cm}^3}{\text{s}} = 0.403 \frac{\text{in}^3}{\text{s}} = \frac{\pi}{4} (0.75)^2 V_1,$$

$$V_1 = \mathbf{0.91 \frac{\text{in}}{\text{s}}} \quad \text{Ans. (b)}$$

3.24 Water enters the bottom of the cone in the figure at a uniformly increasing average velocity $V = Kt$. If d is very small, derive an analytic formula for the water surface rise $h(t)$, assuming $h = 0$ at $t = 0$.



Solution: For a control volume around the cone, the mass relation becomes

$$\frac{d}{dt} \left(\int \rho \, dv \right) - \dot{m}_{in} = 0 = \frac{d}{dt} \left[\rho \frac{\pi}{3} (h \tan \theta)^2 h \right] - \rho \frac{\pi}{4} d^2 Kt$$

$$\text{Integrate: } \rho \frac{\pi}{3} h^3 \tan^2 \theta = \rho \frac{\pi}{8} d^2 Kt^2$$

$$\text{Solve for } \mathbf{h(t) = \left[\frac{3}{8} Kt^2 d^2 \cot^2 \theta \right]^{1/3}} \quad \text{Ans.}$$

3.25 As will be discussed in Chaps. 7 and 8, the flow of a stream U_0 past a blunt flat plate creates a broad low-velocity wake behind the plate. A simple model is given in Fig. P3.25, with only half of the flow shown due to symmetry. The velocity profile behind the plate is idealized as “dead air” (near-zero velocity) behind the plate, plus a higher