velocity, decaying vertically above the wake according to the variation $u \approx U_o + \Delta U e^{-z/L}$, where L is the plate height and $z = 0$ is the top of the wake. Find ΔU as a function of stream speed U_o.

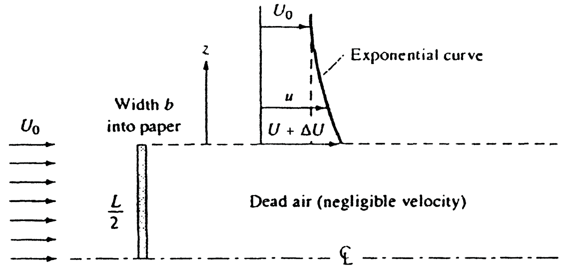


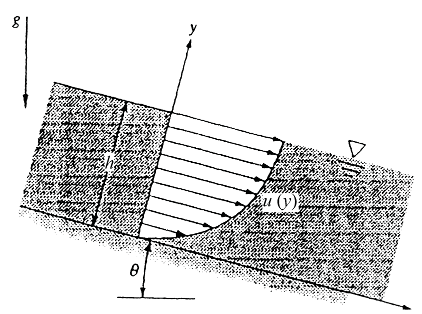
Fig. P3.25

Solution: For a control volume enclosing the upper half of the plate and the section where the exponential profile applies, extending upward to a large distance H such that exp(–H/L) ≈ 0, we must have inlet and outlet volume flows the same:

$$Q_{in} = \int_{-L/2}^{H} U_o\,dz = Q_{out} = \int_0^H (U_o + \Delta U\,e^{-z/L})\,dz, \quad \text{or:} \quad U_o\left(H + \frac{L}{2}\right) = U_oH + \Delta UL$$

$$\text{Cancel } U_oH \text{ and solve for} \quad \Delta U \approx \mathbf{\frac{1}{2}U_o} \quad \textit{Ans.}$$

3.26 A thin layer of liquid, draining from an inclined plane, as in the figure, will have a laminar velocity profile $u = U_o(2y/h - y^2/h^2)$, where U_o is the surface velocity. If the plane has width b into the paper, (a) determine the volume rate of flow of the film. (b) Suppose that h = 0.5 in and the flow rate per foot of channel width is 1.25 gal/min. Estimate U_o in ft/s.



Solution: (a) The total volume flow is computed by integration over the flow area:

$$Q = \int V_n\,dA = \int_0^h U_o\left(\frac{2y}{h} - \frac{y^2}{h^2}\right)b\,dy = \mathbf{\frac{2}{3}U_obh} \quad \textit{Ans. (a)}$$

(b) Evaluate the above expression for the given data:

$$Q = 1.25\ \frac{gal}{\min} = 0.002785\ \frac{ft^3}{s} = \frac{2}{3}U_obh = \frac{2}{3}U_o(1.0\,ft)\left(\frac{0.5}{12}\,ft\right),$$

$$\textit{solve for} \quad U_o = \mathbf{0.10\ \frac{ft}{s}} \quad \textit{Ans. (b)}$$
