$$\text{or:}\quad F_o = 2\rho_o A_o V_o^2, \quad \text{solve for } V_o = \sqrt{\frac{F_o}{2\rho_o(\pi/4)D_o^2}} \quad \textit{Ans.}$$

3.42 A liquid of density ρ flows through the sudden contraction in Fig. P3.42 and exits to the atmosphere. Assume uniform conditions (p_1, V_1, D_1) at section 1 and (p_2, V_2, D_2) at section 2. Find an expression for the force F exerted by the fluid on the contraction.

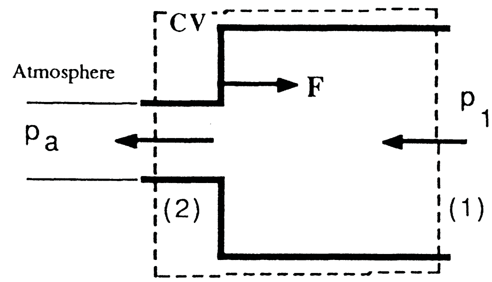

Fig. P3.42

Solution: Since the flow exits directly to the atmosphere, the exit pressure equals atmospheric: $p_2 = p_a$. Let the CV enclose sections 1 and 2, as shown. Use our trick (page 129 of the text) of subtracting p_a everywhere, so that the only non-zero pressure on the CS is at section 1, $p = p_1 - p_a$. Then write the linear momentum relation with x to the right:

$$\sum F_x = F - (p_1 - p_a)A_1 = \dot{m}_2 u_2 - \dot{m}_1 u_1, \quad \text{where } \dot{m}_2 = \dot{m}_1 = \rho_1 A_1 V_1$$

$$\text{But } u_2 = -V_2 \quad \text{and} \quad u_1 = -V_1. \quad \text{Solve for } F_{\underline{\text{on}}\ \text{fluid}} = (p_1 - p_a)A_1 + \rho_1 A_1 V_1(-V_2 + V_1)$$

Meanwhile, from continuity, we can relate the two velocities:

$$Q_1 = Q_2, \quad \text{or} \quad (\pi/4)D_1^2 V_1 = (\pi/4)D_2^2 V_2, \quad \text{or:} \quad V_2 = V_1(D_1^2/D_2^2)$$

Finally, the force of the fluid on the wall is equal and opposite to $F_{\text{on fluid}}$, to the *left*:

$$F_{\text{fluid on wall}} = (p_1 - p_a)A_1 - \rho_1 A_1 V_1^2\left[\left(D_1^2/D_2^2\right) - 1\right], \quad A_1 = \frac{\pi}{4}D_1^2 \quad \textit{Ans.}$$

The pressure term is larger than the momentum term, thus F > 0 and acts to the left.

3.43 Water at 20°C flows through a 5-cm-diameter pipe which has a 180° vertical bend, as in Fig. P3.43. The total length of pipe between flanges 1 and 2 is 75 cm. When the weight flow rate is 230 N/s, p_1 = 165 kPa, and p_2 = 134 kPa. Neglecting pipe weight, determine the total force which the flanges must withstand for this flow.

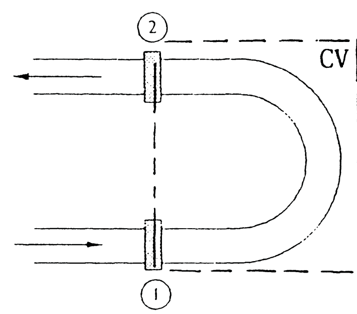

Fig. P3.43