Solution: Let the CV cut through the flanges and surround the pipe bend. The mass flow rate is $(230\text{ N/s})/(9.81\text{ m/s}^2) = 23.45$ kg/s. The volume flow rate is $Q = 230/9790 = 0.0235$ m^3/s. Then the pipe inlet and exit velocities are the same magnitude:

$$V_1 = V_2 = V = Q/A = \frac{0.0235\text{ m}^3\text{/s}}{(\pi/4)(0.05\text{ m})^2} \approx 12.0\ \frac{\text{m}}{\text{s}}$$

Subtract p_a everywhere, so only p_1 and p_2 are non-zero. The horizontal force balance is:

$$\sum F_x = F_{x,flange} + (p_1 - p_a)A_1 + (p_2 - p_a)A_2 = \dot{m}_2u_2 - \dot{m}_1u_1$$

$$= F_{x,fl} + (64000)\frac{\pi}{4}(0.05)^2 + (33000)\frac{\pi}{4}(0.05)^2 = (23.45)(-12.0 - 12.0\text{ m/s})$$

$$\text{or:}\quad F_{x,flange} = -126 - 65 - 561 \approx \mathbf{-750\ N} \quad \textit{Ans.}$$

The total *x*-directed force on the flanges acts to the left. The vertical force balance is

$$\sum F_y = F_{y,flange} = W_{pipe} + W_{fluid} = 0 + (9790)\frac{\pi}{4}(0.05)^2(0.75) \approx \mathbf{14\ N} \quad \textit{Ans.}$$

Clearly the fluid weight is pretty small. The largest force is due to the 180° turn.

3.44 Consider uniform flow past a cylinder with a V-shaped *wake*, as shown. Pressures at (1) and (2) are equal. Let *b* be the width into the paper. Find a formula for the force *F* on the cylinder due to the flow. Also compute $C_D = F/(\rho U^2 Lb)$.

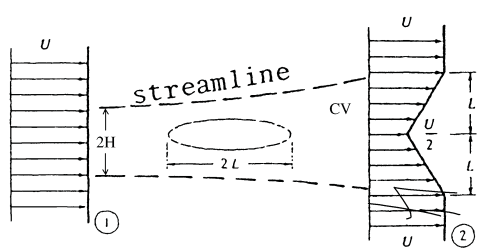

Fig. P3.44

Solution: The proper CV is the entrance (1) and exit (2) plus *streamlines* above and below which hit the top and bottom of the wake, as shown. Then steady-flow continuity yields,

$$0 = \int_2 \rho u\, dA - \int_1 \rho u\, dA = 2\int_0^L \rho \frac{U}{2}\left(1 + \frac{y}{L}\right) b\, dy - 2\rho UbH,$$

where 2H is the inlet height. Solve for H = 3L/4.

Now the linear momentum relation is used. Note that the drag force F is to the right (force of the fluid on the body) thus the force F *of the body on fluid is to the left*. We obtain,

$$\sum F_x = 0 = \int_2 u\rho u\, dA - \int_1 u\rho u\, dA = 2\int_0^L \frac{U}{2}\left(1+\frac{y}{L}\right)\rho\frac{U}{2}\left(1+\frac{y}{L}\right)b\,dy - 2H\rho U^2 b = -F_{drag}$$

$$\text{Use}\quad H = \frac{3L}{4},\quad \text{then}\ F_{drag} = \frac{3}{2}\rho U^2 Lb - \frac{7}{6}\rho U^2 Lb \approx \mathbf{\frac{1}{3}\rho U^2 Lb}\quad \textit{Ans.}$$

The dimensionless force, or drag coefficient $F/(\rho U^2 Lb)$, equals $\mathbf{C_D = 1/3}$. *Ans.*