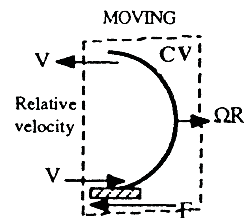


Solution: Let the CV enclose the bucket and jet and let it move to the right at bucket velocity V = ΩR, so that the jet enters the CV at relative speed (Vj – ΩR). Then,

$$\sum F_x = -F_{bucket} = \dot{m}u_{out} - \dot{m}u_{in}$$
$$= \dot{m}[-(V_j - \Omega R)] - \dot{m}[V_j - \Omega R]$$

$$\text{or:}\quad F_{bucket} = 2\dot{m}(V_j - \Omega R) = 2\rho A_j (V_j - \Omega R)^2,$$

$$\text{and the power is}\quad \mathbf{P} = \Omega R F_{bucket} = \mathbf{2\rho A_j \Omega R (V_j - \Omega R)^2} \quad \textit{Ans.}$$

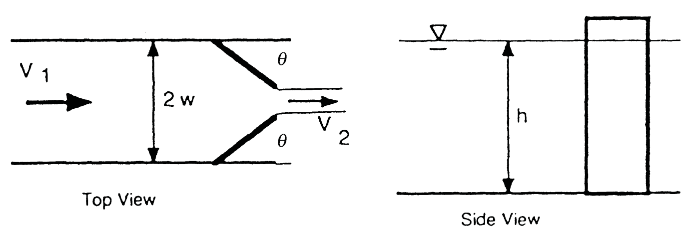
Maximum power is found by differentiating this expression:

$$\frac{dP}{d\Omega} = 0 \quad \text{if} \quad \mathbf{\Omega R = \frac{V_j}{3}} \quad \textit{Ans.} \quad \left(\text{whence } P_{max} = \frac{8}{27}\rho A_j V_j^3\right)$$

If there were many buckets, then the *full* jet mass flow would be available for work:

$$\dot{m}_{available} = \rho A_j V_j, \quad P = 2\rho A_j V_j \Omega R (V_j - \Omega R), \quad \mathbf{P_{max} = \frac{1}{2}\rho A_j V_j^3 \quad at \quad \Omega R = \frac{V_j}{2}} \quad \textit{Ans.}$$

3.52 The vertical gate in a water channel is partially open, as in Fig. P3.52. Assuming no change in water level and a hydrostatic pressure distribution, derive an expression for the streamwise force F_x on one-half of the gate as a function of (ρ, h, w, θ, V_1). Apply your result to the case of water at 20°C, V_1 = 0.8 m/s, h = 2 m, w = 1.5 m, and θ = 50°.



Solution: Let the CV enclose sections (1) and (2), the centerline, and the inside of the gate, as shown. The volume flows are

$$V_1 Wh = V_2 Bh, \quad \text{or:} \quad V_2 = V_1 \frac{W}{B} = V_1 \frac{1}{1 - \sin\theta}$$