Solution: First convert 300 gal/min = 0.01893 m^3/s, hence the mass flow is ρQ = 18.9 kg/s. The vertical-tube velocity (down) is $V_{tube} = 0.01893/[(\pi/4)(0.06)^2] = -6.69$ **k** m/s. The exit tube area is $(\pi/2)R\Delta h = (\pi/2)(0.15)(0.01) = 0.002356\ m^2$, hence $V_{exit} = Q/A_{exit}$ = 0.01893/0.002356 = *8.03* m/s. Now estimate the force components:

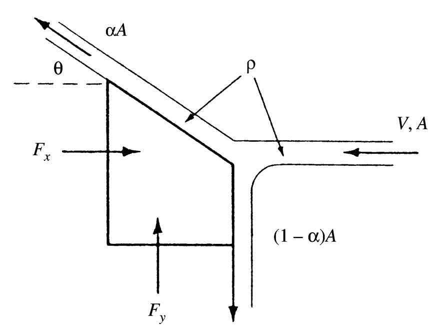
$$\sum F_x = \mathbf{F_x} = \int u_{out}\, d\dot{m}_{out} = \int_{-45^\circ}^{+45^\circ} -V_{exit}\sin\theta\ \rho\Delta hR\, d\theta \equiv \mathbf{0} \quad Ans.\ (a)$$

$$\sum F_y = \mathbf{F_y} = \int v_{out}\, d\dot{m}_{out} - \dot{m}v_{in} = \int_{-45^\circ}^{+45^\circ} -V_{exit}\cos\theta\ \rho\Delta hR\, d\theta - 0 = -V_{exit}\rho\Delta hR\sqrt{2}$$

$$or:\quad \mathbf{F_y} = -(8.03)(998)(0.01)(0.15)\sqrt{2} \approx \mathbf{-17\ N} \quad Ans.\ (b)$$

$$\sum F_z = \mathbf{F_z} = \dot{m}(w_{out} - w_{in}) = (18.9\ kg/s)[0-(-6.69\ m/s)] \approx \mathbf{+126\ N} \quad Ans.\ (c)$$

3.75 A liquid jet of density r and area A strikes a block and splits into two jets, as shown in the figure. All three jets have the same velocity V. The upper jet exits at angle θ and area αA, the lower jet turns down at 90° and area (1 − α)A. (a) Derive a formula for the forces (F_x,F_y) required to support the block against momentum changes. (b) Show that F_y = 0 only if $\alpha \geq 0.5$. (c) Find the values of α and θ for which both F_x and F_y are zero.



Solution: (a) Set up the *x*- and *y*-momentum relations:

$$\sum F_x = F_x = \alpha\dot{m}(-V\cos\theta) - \dot{m}(-V) \quad where\ \ \dot{m} = \rho AV\ \ of\ the\ inlet\ jet$$

$$\sum F_y = F_y = \alpha\dot{m}V\sin\theta + (1-\alpha)\dot{m}(-V)$$

Clean this up for the final result:

$$F_x = \mathbf{\dot{m}V(1-\alpha\cos\theta)}$$
$$F_y = \mathbf{\dot{m}V(\alpha\sin\theta + \alpha - 1)} \quad Ans.\ (a)$$

(b) Examining F_y above, we see that it can be zero only when,

$$\sin\theta = \frac{1-\alpha}{\alpha}$$

But this makes no sense if $\alpha < 0.5$, hence **$F_y = 0$ only if $\alpha \geq 0.5$**. *Ans.* (b)

(c) Examining F_x, we see that it can be zero only if $\cos\theta = 1/\alpha$, which makes no sense unless $\alpha = 1$, $\theta = 0°$. This situation also makes $F_x = 0$ above ($\sin\theta = 0$). Therefore the only scenario for which both forces are zero is the trivial case for which all the flow goes horizontally across a flat block:

$$F_x = F_y = 0 \quad \text{only if: } \alpha = \mathbf{1},\ \theta = \mathbf{0°} \quad \textit{Ans. } (c)$$

3.76 A two-dimensional sheet of water, 10 cm thick and moving at 7 m/s, strikes a fixed wall inclined at 20° with respect to the jet direction. Assuming frictionless flow, find (a) the normal force on the wall per meter of depth, and the widths of the sheet deflected (b) upstream, and (c) downstream along the wall.

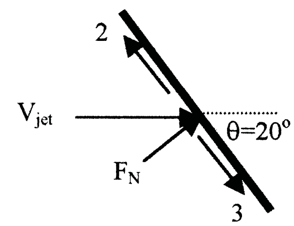


Fig. P3.76

Solution: (a) The force normal to the wall is due to the jet's momentum,

$$\sum F_N = -\dot{m}_{in}u_{in} = -(998)(0.1)(7^2)(\cos 70°) = \mathbf{1670\ N/m} \quad \textit{Ans.}$$

(b) Assuming $V_1 = V_2 = V_3 = V_{jet}$, $V_jA_1 = V_jA_2 + V_jA_3$ where,

$$A_2 = A_1 \sin\theta = (0.1)(1)(\sin 20°) = 0.034 \text{ m} \approx \mathbf{3\ cm} \quad \textit{Ans.}$$

(c) Similarly, $A_3 = A_1 \cos\theta = (0.1)(1)(\cos 20°) = 0.094$ m $\approx$ **9.4 cm** *Ans.*

3.77 Water at 20°C flows steadily through a reducing pipe bend, as in Fig. P3.77. Known conditions are p_1 = 350 kPa, D_1 = 25 cm, V_1 = 2.2 m/s, p_2 = 120 kPa, and D_2 = 8 cm. Neglecting bend and water weight, estimate the total force which must be resisted by the flange bolts.

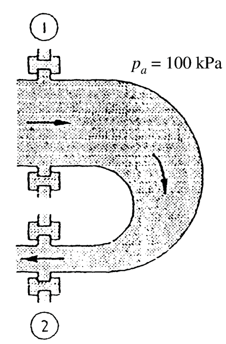


Fig. P3.77

Solution: First establish the mass flow and exit velocity:

$$\dot{m} = \rho_1 A_1 V_1 = 998\left(\frac{\pi}{4}\right)(0.25)^2(2.2) = 108\ \frac{\text{kg}}{\text{s}} = 998\left(\frac{\pi}{4}\right)(0.08)^2 V_2, \quad \text{or} \quad V_2 = 21.5\ \frac{\text{m}}{\text{s}}$$