Solution: Let the CV enclose the cylindrical blob of liquid. With density, area, and the blob volume constant, mass conservation requires that V = V(t) only. The CV accelerates downward at blob speed V(t). Vertical (downward) force balance gives

$$\sum F_{down} - \int a_{rel}\,dm = \frac{d}{dt}\left(\int V_{down}\rho\,d\upsilon\right) + \dot{m}_{out}V_{out} - \dot{m}_{in}V_{in} = 0$$

$$\text{or:}\quad m_{blob}g + \Delta p\,A - \tau_w A_w - a m_{blob} = 0$$

$$\text{Since}\ \Delta p = 0 \quad\text{and}\quad \tau = 0, \quad \text{we are left with}\ \mathbf{a_{blob}} = \frac{\mathbf{dV}}{\mathbf{dt}} = \mathbf{g} \quad \textit{Ans.}$$

3.91 Extend Prob. 3.90 to include a linear (laminar) average wall shear stress of the form $\tau \approx cV$, where c is a constant. Find V(t), assuming that the wall area remains constant.

Solution: The downward momentum relation from Prob. 3.90 above now becomes

$$0 = m_{blob}g - \tau_w \pi DL - m_{blob}\frac{dV}{dt}, \quad \text{or} \quad \frac{dV}{dt} + \zeta V = g, \quad \text{where}\ \zeta = \frac{c\pi DL}{m_{blob}}$$

where we have inserted the laminar shear $\tau = cV$. The blob mass equals $\rho(\pi/4)D^2L$. For V = 0 at t = 0, the solution to this equation is

$$\mathbf{V} = \frac{\mathbf{g}}{\zeta}(\mathbf{1} - \mathbf{e}^{-\zeta t}), \quad \text{where}\ \zeta = \frac{\mathbf{c\pi DL}}{\mathbf{m_{blob}}} = \frac{\mathbf{4c}}{\rho\mathbf{D}} \quad \textit{Ans.}$$

3.92 A more involved version of Prob. 3.90 is the elbow-shaped tube in Fig. P3.92, with constant cross-sectional area *A* and diameter $D \ll h, L$. Assume incompressible flow, neglect friction, and derive a differential equation for *dV/dt* when the stopper is opened. *Hint:* Combine two control volumes, one for each leg of the tube.

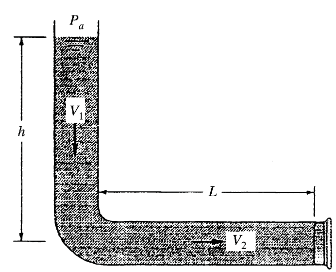

Fig. P3.92

Solution: Use two CV's, one for the vertical blob and one for the horizontal blob, connected as shown by pressure.