Solution: Substitute the given component into continuity and solve for the unknown component:

(a) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0=\frac{\partial}{\partial x}(x^2y)+\frac{\partial v}{\partial y};\ \ \frac{\partial v}{\partial y}=-2xy,$ ***or***: $\mathbf{v=-xy^2+f(x)}$ *Ans.* (a)

(b) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0=\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}(x^2y);\ \ \frac{\partial u}{\partial x}=-x^2,$ ***or***: $\mathbf{u=-\frac{x^3}{3}+f(y)}$ *Ans.* (b)

(c) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0=\frac{\partial}{\partial x}(x^2-xy)+\frac{\partial v}{\partial y};\ \ \frac{\partial v}{\partial y}=-2x+y,$ ***or***: $\mathbf{v=-2xy+\frac{y^2}{2}+f(x)}$ *Ans.* (c)

(d) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0=\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}(y^2-xy);\ \ \frac{\partial u}{\partial x}=-2y+x$ ***or***: $\mathbf{u=-2xy+\frac{x^2}{2}+f(y)}$ *Ans.* (d)

4.11 Derive Eq. (4.12*b*) for cylindrical coordinates by considering the flux of an incompressible fluid in and out of the elemental control volume in Fig. 4.2.

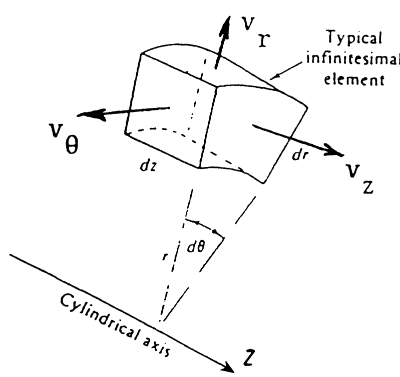

Fig. 4.2

Solution: For the differential CV shown,

$$\frac{\partial\rho}{\partial t}d\upsilon ol+\sum d\dot{m}_{out}-\sum d\dot{m}_{in}=0$$

$$\frac{\partial\rho}{\partial t}\left(r+\frac{dr}{2}\right)d\theta\,dr\,dz+\rho v_r r\,dz\,d\theta+\frac{\partial}{\partial r}(\rho v_r)dr(r+dr)dz\,d\theta+\rho v_\theta\,dz\,dr$$

$$+\frac{\partial}{\partial\theta}(\rho v_\theta)d\theta\,dz\,dr+\rho v_z\left(r+\frac{dr}{2}\right)d\theta\,dr+\frac{\partial}{\partial z}(\rho v_z)\left(r+\frac{dr}{2}\right)d\theta\,dr$$

$$-\rho v_r r\,dz\,d\theta-\rho v_\theta\,dz\,dr-\rho v_z\left(r+\frac{dr}{2}\right)d\theta\,dr=0$$

Cancel ($d\theta drdz$) and higher-order (4th-order) differentials such as ($drd\theta dzdr$) and, finally, divide by *r* to obtain the final result:

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0 \quad \textit{Ans.}$$
